Lecture 2:

- *A very brief history of π.*
  Over 4,000 years of history in 45 minutes!

- *A very brief history of 0.*
  Much ado about nothing
  or
  When did nothing mean something?
π What do you get if you divide the circumference of a bowl of ice cream by its diameter?

π *Pi á la mode!*

π What do you get if you divide the circumference of a jack-o-lantern by its diameter?

π *Pumpkin pi!*

π What do you get if you divide the circumference of an igloo by its diameter?

π *Eskimo pi!*

π “πr²”, said the mathematician.

π “*No!" said the baker, “pies are round!"*

What you probably *do* know ...

- π is the 16th letter of the Greek alphabet.
- π = 3.14159… is a mathematical constant and it’s the ratio of the circumference of a circle to its diameter.
However, what you probably *don’t* know is ...

• a symbol for “π” wasn’t introduced until 1706, by the Welsh mathematician William Jones (1675-1749), and “=” wasn’t in use until introduced by Robert Recorde in 1557!

The true history of π is vague ... but what is certain is that by 2000 BCE, some cultures had grasped the idea that ...

... the wider a circle is “across”, the farther it is “around” ...

... and they had methods for calculating the circumference and area of a circle from its diameter (although they probably did not understand the true significance of π).

A possible translation of a cuneiform tablet from the Old Babylonian period (*ca* 1900-1600 BCE) found near Susa and devoted to geometrical figures, states that

*the ratio of the perimeter of a regular hexagon to the circumference of the circumscribed circle is*

\[
\frac{57}{60} + \frac{36}{(60 \times 60)} = 0.96.
\]

The perimeter of a regular hexagon is 6R and the circumference of the circle is 2πR. So, according to the tablet

\[
\frac{6R}{2\pi R} = \frac{3}{\pi} = 0.96,
\]

giving a value of

\[
\pi = \frac{3}{0.96} = 3.125 \quad (3\frac{1}{8}).
\]
Where might this value of $\pi$ have come from? We have no idea! But what about this suggestion ... the Middle East has ...

a great deal of sand ...

so I wonder if it might have been from “scratching” circles in the sand?

What could be more straightforward than measuring directly the circumference of a circle in terms of its diameter?

Details from OB tablet BM 15285 (drawn by Eleanor Robson). The tablet has over 40 problems mainly dealing with the area of various types of geometrical figures.
Placing a rope of length equal to the diameter in the groove in the sand, they’d find it takes three “lengths” (A → C, C → D, D → E) with a little bit (E → A) left over. Then the question is ...

“what fraction of the diameter is EA?”

The answer is ... between $\frac{1}{7}$ and $\frac{1}{8}$.

$\therefore \pi$ lies between 3.125 and 3.1429.

*However, the Babylonians would have avoided using $\frac{1}{7}$ as it is not a finite sexagesimal fraction, whereas $\frac{1}{8}$ is ($\Rightarrow : 7\ 30$).*

However, there are Babylonian cuneiform tablets of the OB period that put $C = 3 \times D$, i.e., take $\pi = 3$.

Yale Babylonian collection: YBC7302.
Round tablets were usually the work of a student or trainee scribe.

It is conjectured that $3: (= 3)$ is the circumference (C) of the circle, $9: (= 9)$ is $C^2$ and $: 45\ (= 45/60 = 0.75)$ is the area (A). Using our definition for the circumference ($C = 2\pi R$) and area ($A = \pi R^2$), we find

$$\pi = 3.$$
We can get an early Egyptian value for $\pi$ from problems 41 and 48 in the Rhind Mathematical Papyrus, which involve finding the area of a circle from the diameter.

**Problem 41**

Example of making a granary round, of 9 by 10.

Take away thou $\frac{1}{9}$ of 9, namely 1; the remainder is 8.

Multiply 8 up to times 8; there becomes 64.

This is the area of the circular base. In modern notation

$$\text{Area} = \left(9 - 9 \times \frac{1}{9}\right)^2 = 64 = \pi R^2$$

$$\therefore \pi = \frac{64}{8 \times 8} = 3.1605.$$  

The calculation continues for the volume and the amount of grain it contains.
Where did this remarkable idea come from? Probably from problem 48, which is the only example in the papyrus for which the problem is not actually stated!

It consists only of a diagram and a calculation of $8^2$ and $9^2$. The problem is thought to be ...

“It is known that the area of a circle of diameter 9 khet.”

It seems that the area of the circle is approximated by the area of an octagon (an eight-sided figure), viz:

The area of the shaded regions (squares) and hatched (triangles) regions is

$$(5 \times 9) + (4 \times 4 \frac{1}{2}) = 63.$$

The scribe says this is approximately 64, i.e., 8-squared. So, according to the scribe the area of a circle with a diameter of 9 is 64. The area of a circle is $\pi R^2$, so

$$\pi = 3.1605,$$

which is the value used in problem 41.
Greek values for $\pi$ using the method of exhaustion.

- **Antiphon and Bryson of Heraclea** (*ca* 400BCE)

  Calculated the *areas* of inscribed and circumscribed polygons.

- **Archimedes of Syracuse** (*ca* 287-212BCE)

  Improved the method by determining instead the *perimeters* of the inscribed and circumscribed polygons. For 96-sided polygons he found

  $$\frac{22}{7} < \pi < \frac{23}{7} \quad (i.e., \; 3.1408 < \pi < 3.1429)$$

  The average is 3.14185 (~ 0.0082% greater than today’s value).

Biblical value for $\pi$ (from Old Testament passages).

In *1 Kings vii, 23* and *2 Chronicles iv, 2*, we find the following ...

> “Also, he made a molten sea of ten cubits from brim to brim, round in compass, and five cubits the height thereof; and a line of thirty cubits did compass it round about.”
“... ten cubits from brim to brim ... and a line of thirty cubits did compass it round about.”

\[ \therefore \pi = \frac{\text{circumference}}{\text{diameter}} = \frac{\text{distance round}}{\text{width}} = \frac{30 \text{ cubits}}{10 \text{ cubits}} = 3. \]

Is this yet another example in the dispute between science and the biblical record?

Not so fast ...

A few verses later, in 1 Kings vii, 26, it states:

“[the sea] was an hand breadth thick”

One analyst, Rabbi Nehemiah (ca 150 CE), a Hebrew mathematician, claimed the measurement of 10 cubits refers to the outside diameter, while the measurement of 30 cubits is the inside circumference! If one hand breadth = 4”, and one cubit = 18”, then we have:

\[ \pi = \frac{\text{circumference}}{\text{diameter}} = \frac{30 \times 18”}{((10 \times 18”) - (2 \times 4”))}, \]

i.e., \( \pi = 3.1395! \)
Major breakthrough for $\pi$ in the 17th century.

John Wallis (1616-1703) expressed the area of a quadrant of a circle of radius 1 unit as an *infinite* sum of little rectangles. The area of this quadrant is

$$\frac{1}{4}(\pi R^2) = \frac{\pi}{4},$$

since Wallis chose $R = 1$. It was a difficult calculation as calculus hadn’t been invented! After a painful and tedious time involving interpolations and iterations he finally sweated it out:

$$\pi = 2 \cdot \frac{2}{1} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{7} \cdot \frac{6}{8} \cdot \frac{8}{9} \cdots.$$ 

Wallis’s formula (*Arithmetica infinitorum*, 1655).

In Book III (proposition 19, problem 3) of his *Principia Mathematica* (1687), Isaac Newton used $\pi = 3.14159$ to calculate the radius of the Earth.

To get this value for $\pi$ he started with a circle of diameter 1 unit, so the area of the segment ACD is

$$\frac{1}{6}(\pi R^2) = \frac{1}{6} \left( \pi \times \frac{1}{2} \times \frac{1}{2} \right) = \frac{\pi}{24}. $$

Newton calculated $\pi$ correct to the 8th decimal place.
\( \pi \) oddities ...

- Today, \( \pi \) is computed using trigonometric functions. In 1994, the Chudnovsky brothers computed \( \pi \) to over 4 billion places.

- In January 2010 Daisuke Takahashi and his team at Tsukuba University in Japan calculated \( \pi \) to 
  \[ 2,576,980,377,524 \text{ digits} \text{ (~2.6 trillion)} \]
  using a massively parallel computer in 73 h 36 mins. In December 2009, Fabrice Bellard claimed the record at 2,699,999,990,000 digits* using a PC in 131 days. In August 2010, Alexander Yee and Shigeru Kondo claimed 5 trillion digits* using a custom built computer in 90 days.

Why do it?

- *Are the decimal digits uniformly distributed?*

A check of the first trillion decimal places ...

<table>
<thead>
<tr>
<th>Digit</th>
<th>Occurrences</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>99,999,485,134</td>
</tr>
<tr>
<td>1</td>
<td>99,999,945,664</td>
</tr>
<tr>
<td>2</td>
<td>100,000,480,057</td>
</tr>
<tr>
<td>3</td>
<td>99,999,787,805</td>
</tr>
<tr>
<td>4</td>
<td>100,000,357,857</td>
</tr>
<tr>
<td>5</td>
<td>99,999,671,008</td>
</tr>
<tr>
<td>6</td>
<td>99,999,807,503</td>
</tr>
<tr>
<td>7</td>
<td>99,999,818,723</td>
</tr>
<tr>
<td>8</td>
<td>100,000,791,469</td>
</tr>
<tr>
<td>9</td>
<td>99,999,854,780</td>
</tr>
</tbody>
</table>

Total \( 1,000,000,000,000 \)
Can we draw a picture of $\pi$?

Above (left) the first 1600 decimal digits of $\pi$ are displayed in a $(40 \times 40)$ matrix, with a corresponding plot for $22/7$ shown (right). Each white square indicates an even digit and a black square an odd digit.

More $\pi$ oddities ...

- The current record for memorizing the value of $\pi$ is 83,431 places by Akira Haraguchi (a 59 year old mental health counselor) set July 2, 2005. He also claims an unverified record of 100,000 places set October 4, 2006.

- How can you memorize $\pi$? One way is to use a *piem* ...
  ... and what is a piem?

It’s a poem or statement in which the length of the words represent a digit. They are an example of a mnemonic technique to remember the digits of $\pi$, known as *pi-philology*. 
Examples of piems:

Now I, even I, would celebrate.
3 1 4 1 5 9

How I wish I could calculate pi faster.
3 1 4 1 5 9 2 6

How I need a drink, alcoholic of course, after the tough lectures involving quantum mechanics.
3 1 4 1 5 9 2 6 5

All of thy figuring, Prof Jordan, is mighty hard.
3 2 3 8 4 6 2 6 4

Examples in other languages:

Que j’aime à faire apprendre ce nombre utile aux sages! ...
How I would like to learn this number useful to the wise.

Fue y cayó. Y queda solamente la inútil cifra con pocos destinos poderosos, ...
It went and it fell. And only the useless figure remains, with little powerful destinies, ...

Αεί ο Θεός ο Μεγάς γεωμετρεῖ, το κύκλου μήκος ινα οριοθεί διαμέτρῳ, ...
Always the Great God uses Geometry, so that he could define the perimeter of the circle using its diameter, ...
Even more oddities:

• The sequence 0123456789 first appears after 17,387,594,880 digits.

• The sequence 9876543210 first appears after 21,981,157,633 digits.

• If you want to know my birthday, the sequence mmdyyyyy first appears after 117,800,120 digits!

• In the Star Trek episode “Wolf in the Fold” Spock foils an evil computer by commanding it to “compute to last digit the value of π.”.

• March 14 ⇒ 3/14 (in U.S. format) is π day, which is celebrated by many lovers of π! (Incidentally, Albert Einstein was born on π day in 1879.) In Europe ... π Approximation Day is celebrated on July 22 (22/7)!

• Singer, Kate Bush’s 2005 CD “Aerial” contains a song titled “Pi”, in which she sings π to its 137th decimal place (although for some reason, she omits the 79th to 100th decimal places!).

_Ever wonder what π might sound like?

Here is a little “tune” I made up based on the first 33 digits of π, substituting a “white” note for each digit.
Here’s some real *fuzzy math!!*

In 1897, a physician and amateur mathematician from Solitude, Indiana, Edwin J. Goodwin, believed he had discovered a mathematical breakthrough, which he wanted to give as a free gift to the people of Indiana; other States would have to pay royalties if they used it!

The Bill suggests not only one, but *four* values for $\pi$ and an incorrect value for $\sqrt{2}$, viz:

1. $\pi = \frac{16}{\sqrt{3}} = 9.2376…$
   (Surely, the largest overestimate of $\pi$, *EVER!*)

2. $\pi = 2\sqrt{\frac{5\pi}{6}} \Rightarrow 3.236 \times$
   (Should be $\pi = \frac{20}{6} = 3.3\dot{3}$),

3. $\pi = 16\sqrt{\frac{2}{7}} \Rightarrow 3.232,$

4. $\pi = \frac{4}{1.25} \Rightarrow 3.20,$
   and

   $\sqrt{2} = \frac{10}{7} \Rightarrow 1.429.$
So, what happened?

**IN THE HOUSE**

- Read first time January 18, 1897 and referred to the Committee on Canals.
- Reported and referred to Committee on Education January 19, 1897.
- Reported back February 2, 1897.
- Read second time February 5, 1897.
- Ordered engrossed February 5, 1897.
- Read third time February 5, 1897.
  *Passed February 5th, 1897 Ayes: 67 - Noes: 0 !*

**IN THE SENATE**

- Read first time and referred to the Committee on Temperance, February 11, 1897.
- Reported favorable February 12, 1897.
- Read second time and indefinitely postponed February 12, 1897.§.

And one further oddity ...

in 1998, Givenchy launched “π” describing it as

“a woody and magnetic fragrance synonymous of escape and discovery into new sensations. This fresh and masculine scent combines mandarin, iron wood and benzoin crystals bringing a touch of sensuality to the fragrance”. 
So ... when you next have breakfast in a restaurant, ask your server...

“May I have a large container of coffee?

Thank you.”

Smile and remember this talk!!