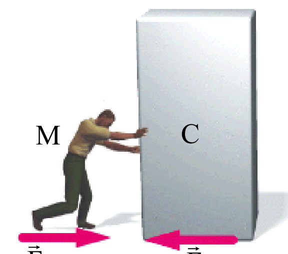
CHAPTER 5

APPLICATIONS OF NEWTON'S LAWS

- Friction
  - static friction
  - kinetic friction
- Circular motion
  - centripetal acceleration
  - centripetal force
  - loop-the-loop
    - airplane
    - roller coaster
- Drag forces

<u>FRICTION</u>

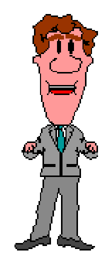
Direction of frictional forces ... (not always obvious) ...



$\vec{F}_{GM}$ frictional force *of the ground on the man*

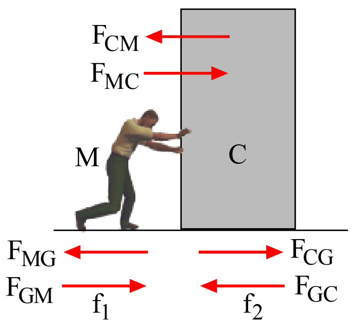
$\vec{F}_{GC}$ frictional force *of the ground on the crate*

Here's the easy way to remember ...



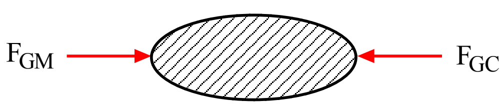
Usually, the frictional force is *opposite* to the direction of the motion *without friction*.

- crate moves to right. $\therefore \vec{F}_{GC} \longleftarrow$
- *without* friction foot would slip to the left. $\therefore \vec{F}_{GM} \longrightarrow$

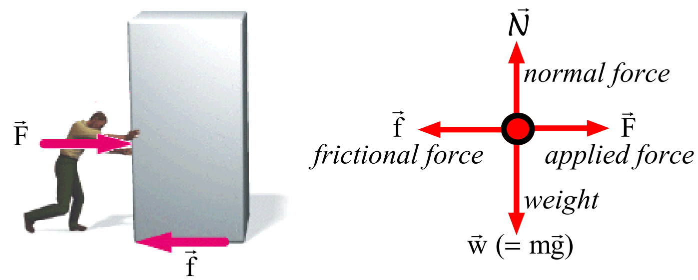


Note that F_{GM} and F_{GC} are frictional forces.

Consider the man and crate as a *system*. The forces F_{MC} and F_{CM} are *internal* to the system; also $|F_{CM}| = |F_{MC}|$, and so they do not contribute to any motion.



The remaining forces acting on the system are F_{GM} and F_{GC}. Thus, if $|F_{GM}| > |F_{GC}|$ the system, i.e., the man and the crate, moves to the right. So, it is the frictional forces f_1 and f_2 that determine the outcome!



Forces acting ON the crate

[1] *Block initially at rest*:

The normal force $\vec{N}$ is always present when an object rests on a surface. You can think of it as a *reaction force* of the surface to the force of the object on the surface. Note

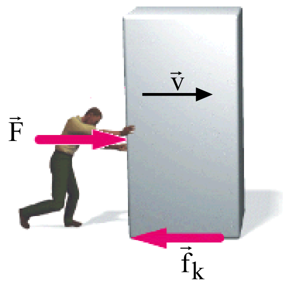
$$\Sigma_y \vec{F}_y = \vec{N} - m\vec{g} = 0, \text{ i.e., } \vec{N} = m\vec{g}.$$

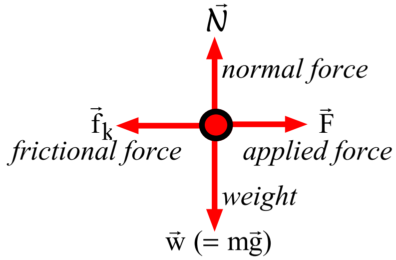
The frictional force that has to be overcome is:

$$|\vec{f}_s| \Rightarrow \mu_s |\vec{N}|,$$

where μ_s is called the *coefficient of static friction*.

If $|\vec{F}| > |\vec{f}_s|$ the block will move. Therefore, a *minimum* applied force $(= \mu_s |\vec{N}|)$ is required to start moving an object over a surface. If $|\vec{F}| \le |\vec{f}_s|$ the *net* force on the block is zero, so the frictional force in this case is $|\vec{f}| = |\vec{F}|$.





Forces acting ON the crate

[2] *Block moving*:

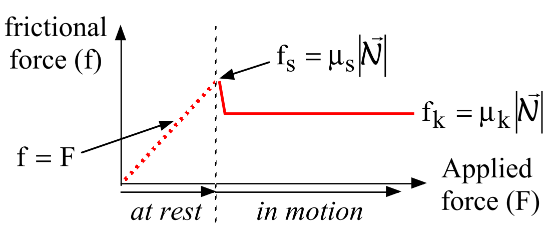
When the object is moving, the frictional force is:

$$|\vec{f}_k| = \mu_k |\vec{N}|,$$

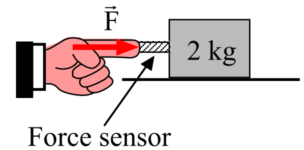
where μ_k is the *coefficient of kinetic friction*. Thus, the net force on the object is then

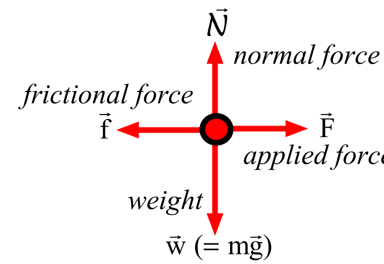
$$|\vec{F}| - |\vec{f}_k|$$

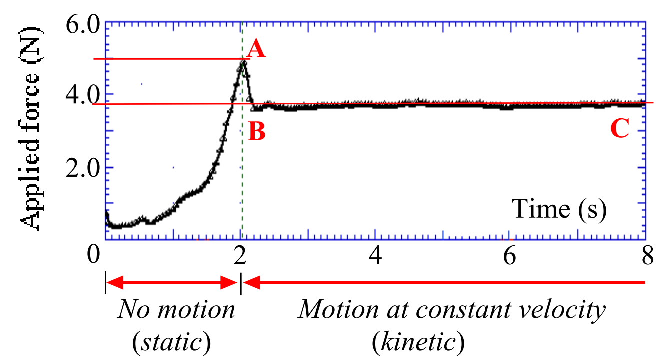
towards the right (= ma). Note, until the applied force $|\vec{F}| > |\vec{f}_s|$ $(= \mu_s |\vec{N}|)$, the *net force* is zero, i.e., $|\vec{F}| = |\vec{f}|$.



Here is some experimental data:







At **A** ("breakout"): $F = f_s = \mu_s N = \mu_s mg$

$$\text{i.e., } \mu_s = F/mg = (5\text{ N})/(2 \times 9.81\text{ m/s}^2) = 0.255.$$

From **B** to **C** (net force = 0): $F = \mu_k N = \mu_k mg$

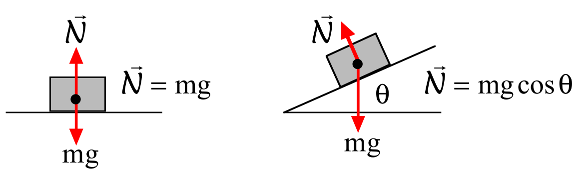
$$\text{i.e., } \mu_k = F/mg = (3.7\text{ N})/(2 \times 9.81\text{ m/s}^2) = 0.189.$$

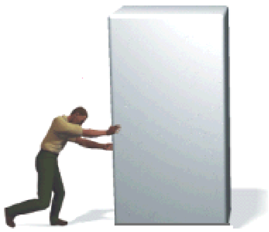
Values of μ_s and μ_k depend on the *two* surfaces in contact:

<u>Materials</u>	μ_s	μ_k
Steel/steel	0.7	0.6
Brass/steel	0.5	0.4
Glass/glass	0.9	0.4
Teflon/steel	0.04	0.04
Rubber/dry concrete	1.0	0.8
Rubber/wet concrete	0.30	0.25
Wax/snow	0.10	0.05

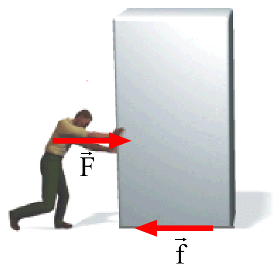
Note: $\mu_s > \mu_k$.

Note also that the NORMAL FORCE $\vec{N}$ is *<u>always</u>* *<u>perpendicular</u>* to the surface *even if the surface is inclined*:





<u>Question 5.1</u>: A horizontal 100 N force exerted on a 1000 N crate causes it to slide across a level floor at constant velocity. What is the magnitude of the frictional force acting on the crate?



Constant velocity means that the net force acting on the crate is zero, i.e.,

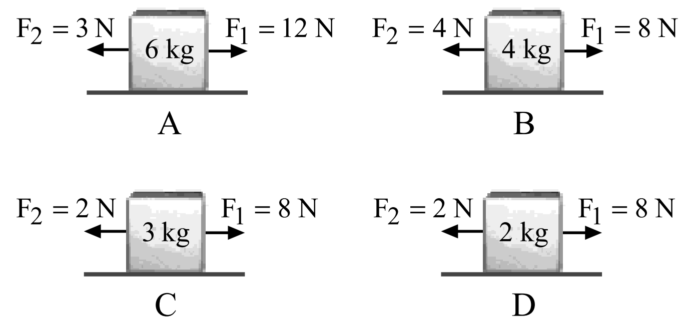
$$\vec{F} + \vec{f} = 0 \Rightarrow \vec{f} = -\vec{F}$$

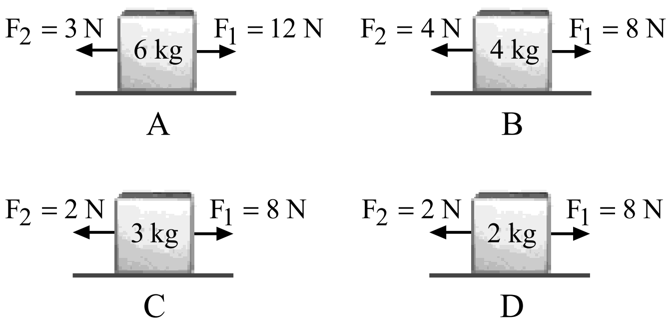
$$\therefore \left|\vec{f}\right| = \left|\vec{F}\right|.$$

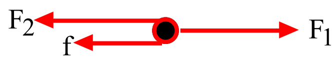
So, the frictional force has the same magnitude as the pushing force although they are in opposing directions, i.e., $\left|\vec{f}\right| = 100$ N.

Do they constitute an action-reaction pair?

<u>Question 5.2</u>: Two forces, F_1 and F_2, act on different objects on a surface, as shown below. If the objects are all moving with constant velocity, in which case is the coefficient of friction between the object and surface greatest?







Draw the FBD. The *net* force on each block is

$$F_1 - F_2 - f = 0$$

since they are moving with constant velocity.

$$\therefore f = F_1 - F_2 = \mu \mathcal{N}, \text{ i.e., } \mu = \frac{(F_1 - F_2)}{\mathcal{N}}.$$

$$\mu_A = {}^{9\text{ N}}\!/_{6\text{g}} = {}^{1.5\text{ N}}\!/_{\text{g}}; \quad \mu_B = {}^{4\text{ N}}\!/_{4\text{g}} = {}^{1\text{ N}}\!/_{\text{g}},$$

smallest

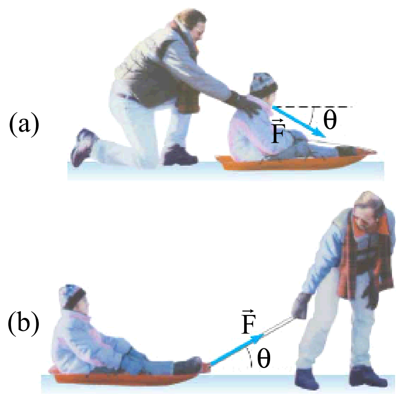
$$\mu_C = {}^{6\text{ N}}\!/_{3\text{g}} = {}^{2\text{ N}}\!/_{\text{g}}; \quad \mu_D = {}^{6\text{ N}}\!/_{2\text{g}} = {}^{3\text{N}}\!/_{\text{g}}.$$

greatest

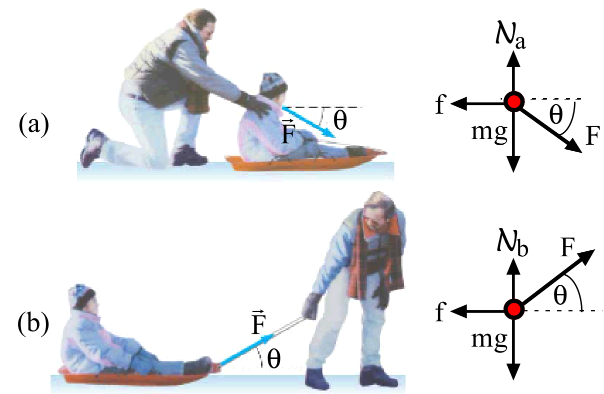
NOTE: the "friction" equation

$$|\vec{f}| = \mu |\vec{\mathcal{N}}|$$

is *not* a vector equation. Even though the coefficient of friction μ is a scalar, $\vec{f}$ is not parallel to $\vec{\mathcal{N}}$.



Question 5.3: Your daughter is sitting on a sled and asks you to propel her across a flat, horizontal surface. You have a choice; (a) you can push on her shoulders, or (b) you can pull her using a rope attached to the sled. Which would require less force from you to get the sled moving, case (a), case (b) or is the force the same in both cases?



Look at the FBD's of the girl/sled system. The minimum force required to start the sled moving in either case is:

$$F_x = F_{min}\cos\theta = \mu_s \mathcal{N}.$$

The angle θ is the same in both cases, so $F_{min} \propto \mathcal{N}$.

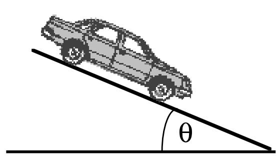
$$\Sigma_a F_y = \mathcal{N}_a - F\sin\theta = mg\text{, i.e., } \mathcal{N}_a = mg + F\sin\theta.$$

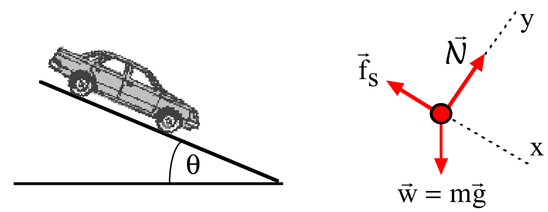
$$\Sigma_b F_y = \mathcal{N}_b + F\sin\theta = mg\text{, i.e., } \mathcal{N}_b = mg - F\sin\theta.$$

$$\therefore \mathcal{N}_a > \mathcal{N}_b.$$

So, $F_{min}(a) > F_{min}(b)$, i.e., a smaller force is required to start the sled moving in (b) than in (a).

Question 5.4: The coefficient of friction between the tires of a car and the road on a particular day is 0.70. What is the steepest slope of the road on which the car is parked if the car is not to slide down the hill? Note, the brakes are fully on so the wheels are locked.





When the car is just about to move:

$$\sum \vec{F}_x = 0 \text{ and } \sum \vec{F}_y = 0.$$

From chapter 4, we have:

- <u>forces along x</u>: $mg\sin\theta - f_s = 0$ ... ... (i),
- <u>forces along y</u>: $\mathcal{N} - mg\cos\theta = 0$ ... ... (ii).

But $f_s = \mu_s \mathcal{N}$.

$$\therefore mg\sin\theta - \mu_s \mathcal{N} = 0.$$

Substituting for $\mathcal{N}$ from (ii) we get:

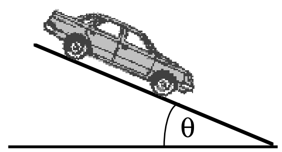
$$mg\sin\theta - \mu_s mg\cos\theta = 0.$$

$$\boxed{\therefore \tan\theta = \mu_s.}$$

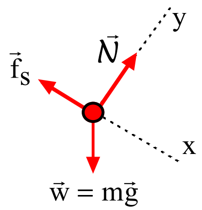
So, at this angle θ the car is <u>just</u> about to slide

$$\text{If } \mu_s = 0.70, \text{ then } \theta = \tan^{-1}(0.70) = 35^\circ.$$

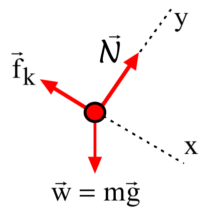
Note: it is independent of the mass of the car and has nothing to do with the "*strength*" of the brakes!



What happens after the car just starts to slide?



Before



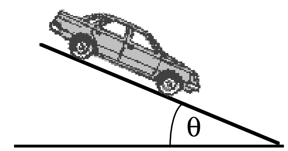
After

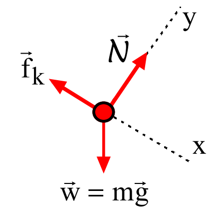
The net force on the car in the x-direction *after* it begins to slide is:

$$F_x = mg\sin\theta - f_k, \text{ where } f_k = \mu_k \mathcal{N}.$$

But $\mu_k < \mu_s$, so $f_k < f_s$, which means that after the car begins to slide, $F_x > 0$, i.e., *there is now a net force in the x-direction*. By Newton's 2nd Law the car will accelerate in the same direction as that force, i.e., down the incline.

Let us calculate the acceleration assuming $\mu_k = 0.50$.





After the car starts to move we replace $\vec{f}_s$ by $\vec{f}_k$ and by Newton's 2nd Law, the net force is ma_x, where a_x is the acceleration of the car.

So:

<u>forces along x</u>: $mg\sin\theta - f_k = ma_x$ ... ... (iii)

with $f_k = \mu_k \mathcal{N}$.

<u>forces along y</u>: $\mathcal{N} - mg\cos\theta = 0$ (*unchanged*)

Then (iii) becomes: $ma_x = mg\sin\theta - \mu_k mg\cos\theta$.

$$\text{i.e., } a_x = g(\sin\theta - \mu_k\cos\theta).$$

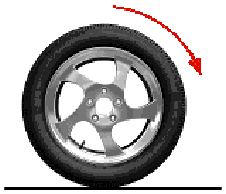
In this case (with $\mu_k = 0.50$):

$$\therefore a_x = \left(9.81 \text{ m/s}^2\right) \times (\sin 35^\circ - 0.50\cos 35^\circ)$$

$$= 1.61 \text{ m/s}^2.$$

Wheels on surfaces:

Two possibilities:



- rolling wheel - no slipping - *STATIC FRICTION*
- locked wheel - skidding - *KINETIC FRICTION*

Since $\mu_k < \mu_s$ then $f_k < f_s$ and so there is less frictional force in a skid $\Rightarrow$ you travel further!

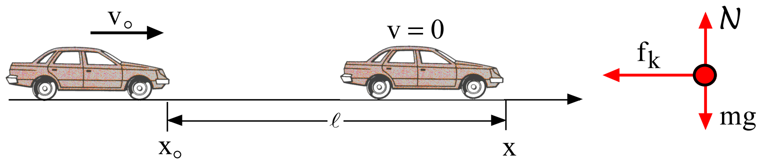
Look at the web-site $\Rightarrow$ *anti-lock (abs) brakes*

Question 5.5: The driver of a 1200 kg car moving at 15.0 m/s is forced to slam on the brakes. The car skids to a halt after traveling a distance of 25.5 m.

(a) What is the coefficient of kinetic friction between the road and the tires?

(b) How long did it take the car to stop?

(c) If the car had been fitted with an anti-skid control system (ABS) and the coefficient of static friction between the road and the tires was $\mu = 0.65$, how far would the car have traveled before it came to a complete stop?



(a) Find the acceleration. We have $0 = v_\circ^2 + 2a(x - x_\circ)$,

$$\text{i.e., } a = -\frac{v_\circ^2}{2\ell} = -\frac{(15.0\ \text{m/s})^2}{2(25.5\ \text{m})} = -4.41\ \text{m/s}^2.$$

The only force in the x-direction is the kinetic frictional force, that is the *net* force, so, by Newton's 2nd Law:

$$-f_k = ma = -\mu_k \mathcal{N} = -\mu_k mg$$

$$\therefore \mu_k = \frac{-a}{g} = \frac{-(-4.41\ \text{m/s}^2)}{9.81\ \text{m/s}^2} = 0.45.$$

Note that the result is independent of the mass of the car!

(b) We have $v = v_\circ + at$,

$$\text{i.e., } t = \frac{v - v_\circ}{a} = \frac{-15.0\ \text{m/s}}{-4.41\ \text{m/s}^2} = 3.40\ \text{s}.$$

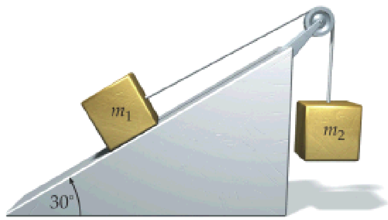
(c) In part (a) we found $a = -\dfrac{v_\circ^2}{2\ell}$.

Also $-f_k = ma = -\mu_s \mathcal{N} = -\mu_s mg$,

$$\text{i.e., } a = -\mu_s g = -\frac{v_\circ^2}{2\ell}.$$

$$\therefore \ell = \frac{v_\circ^2}{2\mu_s g} = \frac{(15.0\ \text{m/s})^2}{2(0.65)(9.81\ \text{m/s}^2)} = 17.6\ \text{m}.$$

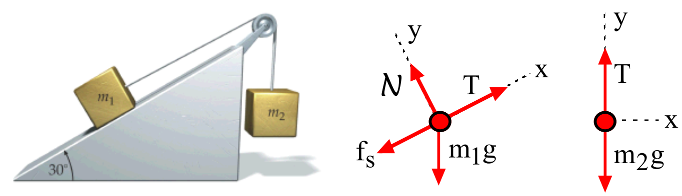
This distance is 7.9 m less, i.e., 31% less, than the result in part (b). The moral is when coming to a stop, try not to skid!



Question 5.6: In the figure above, $m_1 = 4$ kg and the coefficient of static friction between the block and the inclined surface is 0.40. (a) Find the range of possible values of m_2 for which the blocks are stationary, i.e., in equilibrium. (b) What is the frictional force on m_1 if $m_2 = 1$ kg and in which direction does it act?

We need to analyze two scenarios; [1] when m_1 is about to *move up the slope*, and [2] when m_1 is about to *move down the slope*. *WHY??*

[1] Assume m_1 is just about to move <u>up</u> the slope:



$$\Sigma_1 F_x = T + (-f_s) + (-m_1 g \sin 30^\circ) = 0 \quad \ldots \quad (1)$$

$$\Sigma_1 F_y = \mathcal{N} + (-m_1 g \cos 30^\circ) = 0 \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\Sigma_2 F_y = T + (-m_2 g) = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

With $m_1 = 4$ kg and $\mu_s = 0.40$, eq (2) gives $\mathcal{N} = 34.0$ N.

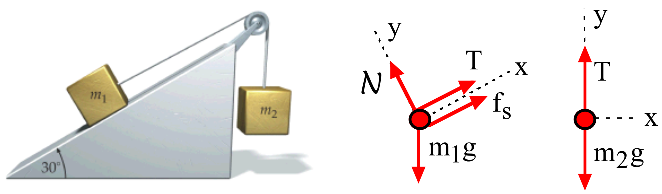
But $f_s = \mu_s \mathcal{N} = 0.40 \times 34.0 \text{ N} = 13.6 \text{ N}$.

From (1): $T = f_s + m_1 g \sin 30^\circ$

$$= 13.6 \text{ N} + \left(4 \text{ kg} \times 9.81 \text{ m/s}^2 \times 0.50\right) = 33.2 \text{ N},$$

so from (3): $$m_2 = \frac{T}{g} = \frac{33.2 \text{ N}}{9.81 \text{ m/s}^2} = 3.39 \text{ kg}.$$

[2] Assume m_1 is just about to move <u>down</u> the slope:



Now:

$$\Sigma_1 F_x = T + f_s + (-m_1 g \sin 30^\circ) = 0 \quad \ldots \quad \ldots \; (4)$$

$$\Sigma_1 F_y = \mathcal{N} + (-m_1 g \cos 30^\circ) = 0 \quad \ldots \quad (2) \; \textit{unchanged.}$$

$$\Sigma_2 F_y = T + (-m_2 g) = 0 \quad \ldots \quad \ldots \quad \ldots \quad (3) \; \textit{unchanged.}$$

With $m_1 = 4$ kg and $\mu_s = 0.40$, eq (2) gives $\mathcal{N} = 34.0$ N.

But $f_s = \mu_s \mathcal{N} = 0.40 \times 34.0 \text{ N} = 13.6 \text{ N}$.

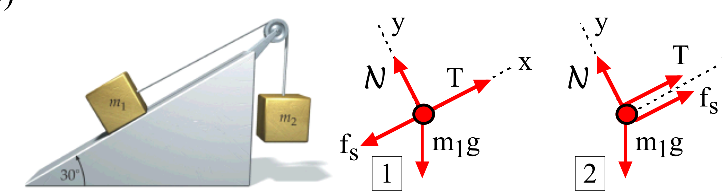
From (4): $T = -f_s + m_1 g \sin 30^\circ$

$$= -13.6 \text{ N} + \left(4 \text{ kg} \times 9.81 \text{ m/s}^2 \times 0.50\right) = 6.0 \text{ N}.$$

so from (3): $m_2 = \dfrac{T}{g} = \dfrac{6.0 \text{ N}}{9.81 \text{ m/s}^2} = 0.61 \text{ kg}$.

Thus, m_1 remains stationary if $0.61 \text{ kg} < m_2 < 3.39 \text{ kg}$.

(b)



If $m_2 = 1$ kg, eq (3) gives, $T = m_2 g = 9.81$ N.

Since $0.6\text{kg} < m_2 < 3.39\text{kg}$, m_1 is stationary. The net force acting on m_1 is:

$$\Sigma F_x = T \pm f_s + (-m_1 g \sin 30^\circ) = 0,$$

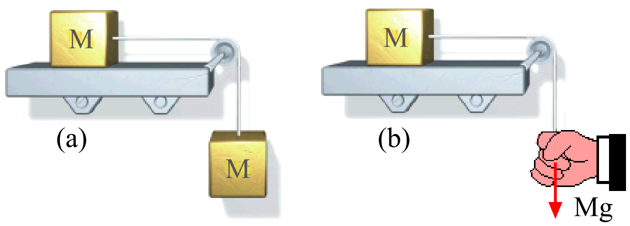
i.e., $\quad T \pm f_s = m_1 g \sin 30^\circ$

$$= 4 \text{ kg} \times 9.81 \text{ m/s}^2 \times 0.50 = 19.62 \text{ N}.$$

But, in which direction is the static frictional force f_s?

Since $T = 9.81$ N then $f_s = 9.81$ N, so they act in the <u>*same*</u> direction, i.e., f_s is directed *up* the slope and together they 'balance' the component of the weight $\left(m_1 g \sin 30^\circ\right)$ acting down the slope.

DISCUSSION PROBLEM [5.1]:



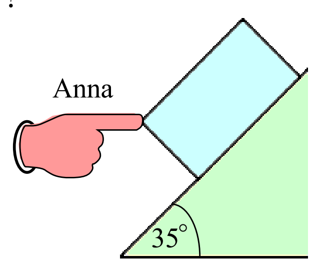
Equal downward forces ($F = Mg$) are applied to each string. In case (a) the force is supplied by a hanging mass M, in case (b) the force is supplied by a hand pulling the string. If the coefficient of static friction is < 1, the blocks on the table will accelerate. In which case is the acceleration of the block on the table greater?

A: Case (a).

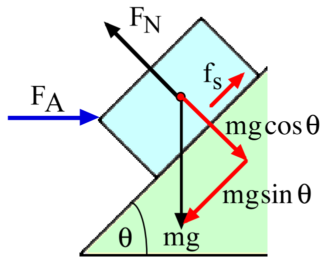
B: Case (b).

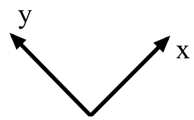
C: Neither ... it's the same in both cases.

<u>Question 5.7</u>: (Similar to question 4.6 in the previous chapter but with friction added.) To prevent a box from sliding down an inclined plane, physics student Anna pushes on the box horizontally with just enough force so that the box is stationary. If the mass of the box is 2.00 kg, the slope of the incline is 35° and the coefficient of static friction between the box and the incline is $\mu_s = 0.160$, what is the magnitude of the minimum force she has to apply?

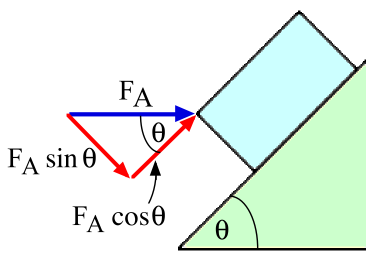


Draw the free body diagram for the box:





The components of the weight force along x and along y are $(-mg\sin\theta)$ and $(-mg\cos\theta)$, respectively. The components of the force Anna applies (F_A) along x and y



are $F_A \cos\theta$ and $(-F_A \sin\theta)$, respectively. Summing the forces in the x- direction, we have

$$\sum F_x = 0$$

$$\text{i.e., } F_A \cos\theta + f_s - mg\sin\theta \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

with $f_s = \mu_s F_N$.

In the y-direction we have:

$$-F_A \sin\theta + F_N - mg\cos\theta = 0,$$

$$\text{i.e., } \quad F_N = mg\cos\theta + F_A \sin\theta.$$

Substituting for F_N in eq (1), we get

$$F_A \cos\theta + \mu_s(mg\cos\theta + F_A \sin\theta) - mg\sin\theta = 0.$$

Solving for F_A, we find

$$F_A \cos\theta + \mu_s F_A \sin\theta = mg\sin\theta - \mu_s mg\cos\theta,$$

$$\text{i.e., } \quad F_A = \frac{mg(\sin\theta - \mu_s \cos\theta)}{(\cos\theta + \mu_s \sin\theta)}.$$

Inserting the given values

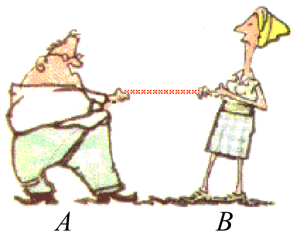
$$F_A = \frac{2.00 \text{ kg} \times 9.81 \text{ m/s}^2 \times (0.574 - 0.160 \times 0.819)}{(0.819 + 0.160 \times 0.574)}$$

$$= 9.54 \text{ N}.$$

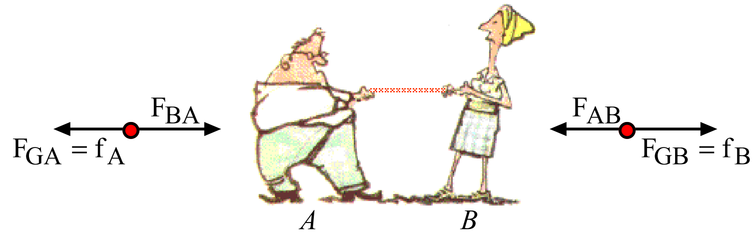
In chapter 4, without friction $(\mu_s = 0)$ we found

$$F_A = mg\tan\theta = 13.7 \text{ N}.$$

<u>AT HOME</u>: show that to get the box to just start moving <u>up</u> the incline, Anna would need to apply a horizontal force of 19.0 N.



Question 5.8: Two people (*A* and *B*) are tugging at each other. By Newton's 3rd law the force that *A* applies on *B* is equal to the force that *B* applies on *A*. How come one of them can win this tug-of-war?



Identify the forces acting *on* each person. F_{AB} and F_{BA} are an A/R pair.

$$\therefore F_{AB} = F_{BA}.$$

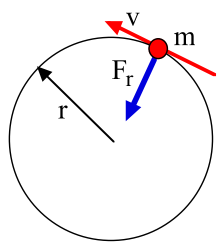
Note that the (frictional) forces are one part of A/R pairs also, i.e., $F_{GA} = F_{AG}$ and $F_{GB} = F_{BG}$

Since F_{AB} and F_{BA} *cancel* each other the result depends on the frictional forces F_{GA} and F_{GB}.

- If $F_{GA} > F_{GB}$ then A wins!
- If $F_{GB} > F_{GA}$ then B wins!

If $f = \mu_s N$ for both people then the person with greater mass should win! Note that if there's no friction (e.g., on ice) then $F_{GA} = F_{GB} = 0$ so no one wins!

CIRCULAR MOTION



In chapter 3, we found that in the case of an object traveling with a constant speed v in a circle of radius r, the object experiences a centripetal acceleration directed towards the center of the circle. The magnitude of the acceleration is

$$a_r = \frac{v^2}{r}.$$

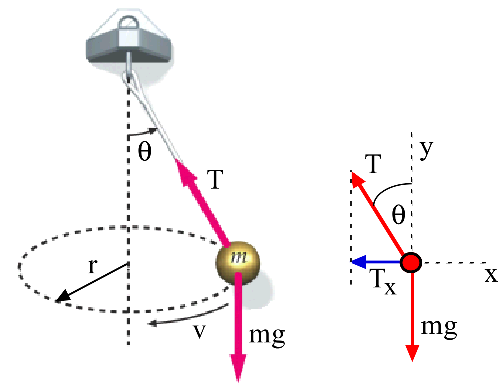
The corresponding radial force acting on the object is

$$F_r = ma_r = m\frac{v^2}{r},$$

which is called the *centripetal* force. In the absence of this force, the object would continue in a straight line (Newton's 1st Law). Examples of centripetal force include the action of the tension in a string spinning a stone in a circle and the gravitational force between the Earth and Moon.

However, there is a difference between *horizontal* and *vertical* circular motion.

[1] *Horizontal circular orbit*

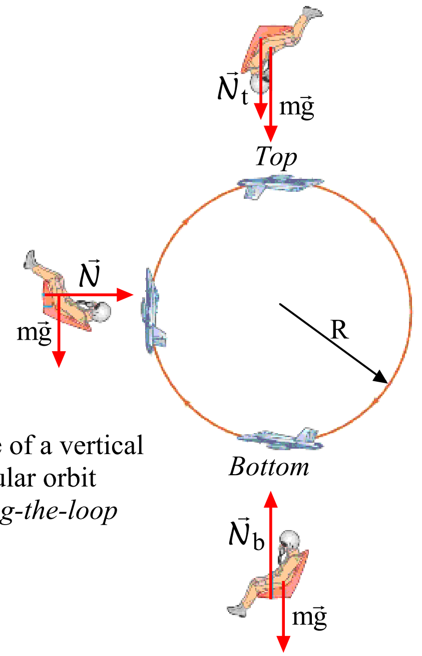


Horizontal circular orbit
conical pendulum

The weight force (mg) is always perpendicular to the plane of the motion and so it cannot produce the centripetal force. It is the x−component (horizontal) of the tension that produces the centripetal force.

$$\text{i.e.,} \qquad T_x = T\sin\theta = F_r = m\frac{v^2}{r}.$$

[2] *Vertical circular orbit*

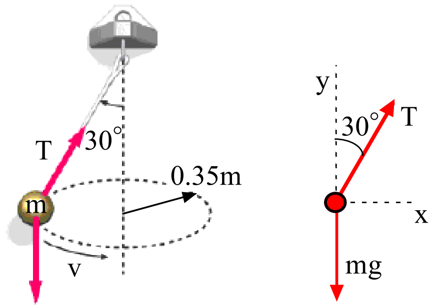


Example of a vertical circular orbit
looping-the-loop

The weight force (mg) and normal force (N) of the seat on the pilot combine to produce the centripetal force on the pilot. Note that the direction of the normal force changes around the loop.

Question 5.9: A stone, of mass 0.75 kg, is attached to a string and whirled in a horizontal circle of radius 35 cm (like a conical pendulum). If the string makes an angle of 30° with the vertical, find (a) the tension in the string, (b) the speed of the stone and (c) the time for one revolution, i.e., the orbital period.

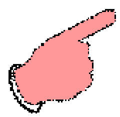
(a)



In the y−direction: $T\cos 30^{\circ} + (-mg) = 0$ ... (i)

In the x−direction: $T\sin 30^{\circ} = ma_x$ ... ... (ii)

centripetal force



From earlier, the centripetal acceleration $a_x = v^2/r$.

$$\text{From (i):}\quad T = \frac{mg}{\cos 30^{\circ}} = 8.50\text{ N.}$$

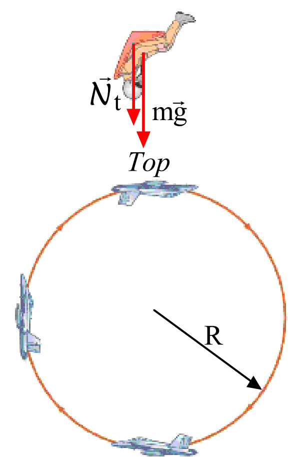
(b) $$\text{From (ii):}\quad a_x = \frac{T\sin 30^{\circ}}{m} = 5.66\text{ m/s}^2 = v^2/r\,.$$

$$\therefore v = \sqrt{ra_x} = 1.41\text{ m/s.}$$

(c) $$\text{Orbital time} = 2\pi r/v = 1.56\text{ s.}$$

Analysis of the forces acting on a pilot during a loop.

1. At the top (inside the loop)



At the top the two forces acting <u>*on*</u> the pilot are the normal force ($\vec{N}_t$) due to the seat, and the gravitational force (mg) due to the Earth. In executing the maneuver, the net downward force produces the centripetal force, i.e.,

$$F_{top} = N_t + mg = \frac{mv^2}{R}.$$

So, the *magnitude* of the force of the seat on the pilot, i.e., the nomal force, is:

$$N_t = \frac{mv^2}{R} - mg\,.$$

Since $$\mathcal{N}_t = \frac{mv^2}{R} - mg,$$

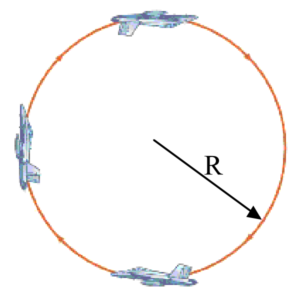
there are two possibilities to consider:

• If $\frac{mv^2}{R} \geq mg$, i.e., $v \geq \sqrt{gR}$, then $\mathcal{N}_t > 0$, so the seat applies a force on the pilot, but it is less than his *true weight* (mg). By Newton's 3rd Law this is the same as the force exerted by the pilot on the seat. Therefore, the pilot experiences an *apparent weight* ($\mathcal{N}_t$) that is less than mg.

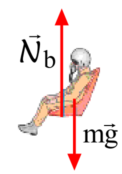
• If $\frac{mv^2}{R} < mg$, i.e., $v < \sqrt{gR}$, then $\mathcal{N}_t < 0$, so the pilot will 'fall' from the seat unless restrained by a seat-belt! (The seat can only provide a *positive* force.)

Thus, if unrestrained, whether or not the pilot remains in the seat depends on the speed of the airplane.

2. At the bottom (inside the loop):



Bottom



At the bottom, the free body diagram tells us the centripetal force is:

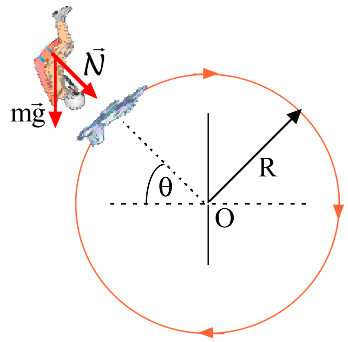
$$\frac{mv^2}{R} = \mathcal{N}_b - mg.$$

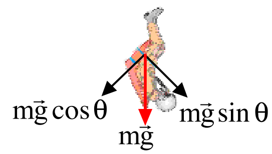
So, the *magnitude* of the force exerted by the seat on the pilot is

$$\mathcal{N}_b = mg + \frac{mv^2}{r}$$

which is, clearly, greater than mg, the *true weight* of the pilot. But, by the 3rd Law, $\mathcal{N}_b$ is also the magnitude of the force the pilot exerts on the seat, i.e., the *apparent weight* of the pilot. Thus, his apparent weight is greater than his true weight.

3. At some angle (inside the loop):





components of the pilot's weight

In the radial direction we have

$$\sum F_R = \frac{mv^2}{R} = N + mg\sin\theta,$$

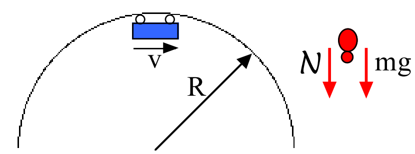
where v is the instantaneous speed. After re-arranging, we obtain

$$N = m\left(\frac{v^2}{R} - g\sin\theta\right).$$

Note, if we put $\theta = 90^\circ$ (top) and $\theta = 270^\circ$ (bottom) we obtain the previous results.

Analysis of the forces acting on the riders on a roller coaster ride doing a loop:

3. At the top (inside the loop)



The analysis is the same as case #1, i.e., the centripetal force experienced by the rider is

$$\frac{mv^2}{R} = N + mg,$$

and so the force applied by the seat on the rider is

$$N = \frac{mv^2}{R} - mg.$$

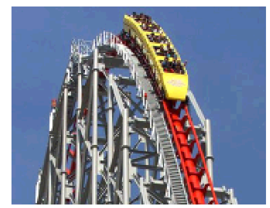
So, if $mg > \frac{mv^2}{R}$, $\mathcal{N} < 0$, and the riders will "fall" from their seat (as a seat can only provide a *positive* force!) Consequently, there is a minimum speed (v_{min}) for a roller coaster to carry out this maneuver safely, i.e.,

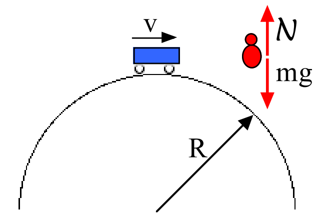
$$v_{min} = \sqrt{Rg}.$$

With a speed *less than this value*, the riders *will fall from their seats* unless restrained. (Note, it does not depend on the rider's mass, i.e., it is the same for all riders!)

In fact, if $v_{min} < \sqrt{Rg}$, the roller coaster cars will fall also unless the track is designed to hold onto the cars!

4. At the top (outside the loop)





The free body diagram gives:

$$\frac{mv^2}{R} = mg - \mathcal{N},$$

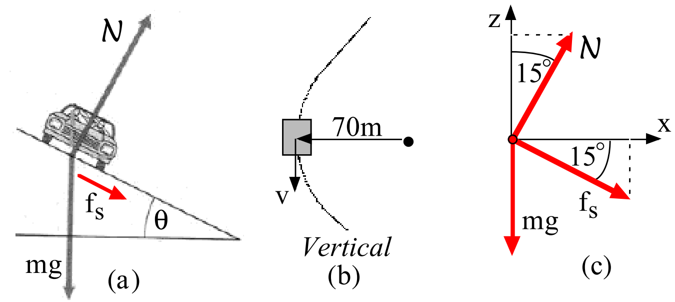
so the force applied by the seat on the rider is

$$\mathcal{N} = mg - \frac{mv^2}{R}.$$

Providing $v < \sqrt{Rg}$, then $\mathcal{N} > 0$, so the rider remains in their seat. However, if $v > \sqrt{Rg}$, then $\mathcal{N} < 0$ and the rider "flies" from their seat (unless restrained)! Thus, there is a maximum speed for this maneuver to be safely executed.

Question 5.10: A curve in the road, with a radius of 70.0 m, is banked at an angle of 15°. If the coefficient of friction between the road and car tires is 0.70, what is the maximum speed a car can make the corner without sliding?

Hint: eliminate the normal force.



Figures (a) and (b) show the *real life* scenario; (c) is the free body diagram of (a). Note that the frictional force f_s is a *static* frictional force as there is no relative motion between the tire and the road, i.e., no skidding or wheel spin. Take *radial* (x) and *vertical* (z) components in (c). Note: the net radial force is the centripetal force. In (a),

<u>*x-direction*</u>: $\mathcal{N}\sin 15^\circ + f_s \cos 15^\circ = mv^2/r$.

<u>*z-direction*</u>: $\mathcal{N}\cos 15^\circ - mg - f_s \sin 15^\circ = 0.$

But $f_s = \mu_s \mathcal{N}$. So, re-arranging these equations we get:

<u>*x-direction*</u>: $\mathcal{N}\sin 15^\circ + \mu_s \mathcal{N}\cos 15^\circ = mv^2/r$.

<u>*z-direction*</u>: $\mathcal{N}\cos 15^\circ - \mu_s \mathcal{N}\sin 15^\circ = mg$.

Hence $\quad \mathcal{N}\left(\sin 15^\circ + \mu_s \cos 15^\circ\right) = mv^2/r \quad \ldots \quad \ldots \quad [1]$

and $\quad \mathcal{N}\left(\cos 15^\circ - \mu_s \sin 15^\circ\right) = mg \quad \ldots \quad \ldots \quad \ldots \quad [2].$

Dividing eq. [1] by eq. [2] to cancel $\mathcal{N}$, we get

$$\frac{\sin 15^\circ + \mu_s \cos 15^\circ}{\cos 15^\circ - \mu_s \sin 15^\circ} = \frac{v^2}{gr}.$$

i.e., $\quad v^2 = gr\left(\dfrac{\tan 15^\circ + \mu_s}{1 - \mu_s \tan 15^\circ}\right)$

$$= \left(9.81\ \text{m/s}^2 \times 70\ \text{m}\right)\left(\frac{0.268 + 0.70}{1 - (0.70 \times 0.268)}\right).$$

$$= 818.2\ (\text{m/s})^2 \Rightarrow v = 28.6\ \text{m/s}.$$

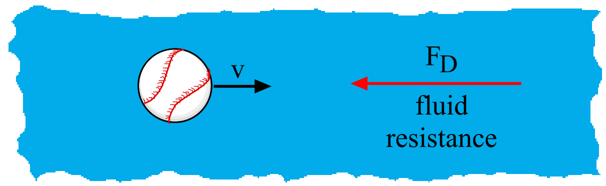
Note that the result is independent of the mass of the car!

At home, show that:

(1) if $\theta = 0$ and $\mu_s = 0.70$, i.e., no banking but static friction, then $v = 21.9$ m/s.

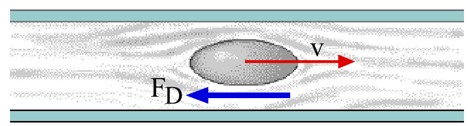
(2) if $\theta = 15^\circ$ and $\mu_s = 0$, i.e., banked corner but zero static friction (like ice), then $v = 13.6$ m/s.

DRAG FORCES



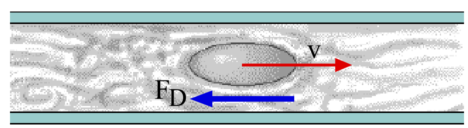
Different from "ordinary" friction because it depends on speed and type of "flow":

- at low speed (non-turbulent or laminar flow)



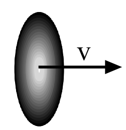
$F_D = kv$

- at high speed (turbulent flow)

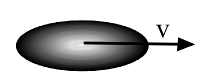


$F_D = bv^2$

The constants depend on shape, size, etc. For example:

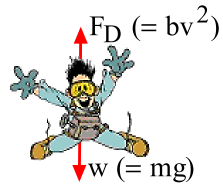


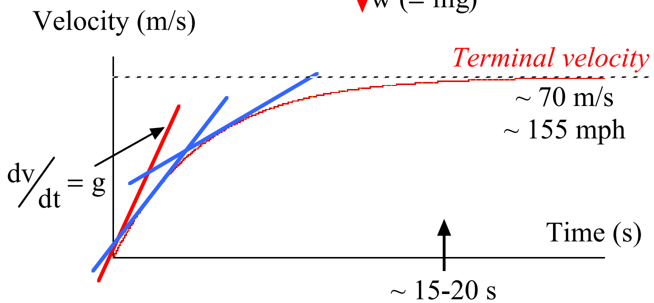
Large b (large area)



Small b (small area)

<u>Sky diving</u>





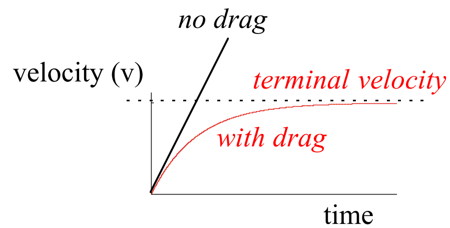
The downward acceleration of a sky diver is gradually reduced because of the upward resistance force due to drag (F_D), which increases with velocity. The net force on the sky-diver is:

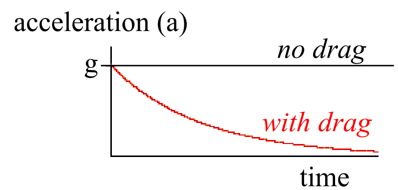
$$F_{net} = w - F_D \;\; (= mg - bv^2) \;\; = ma,$$

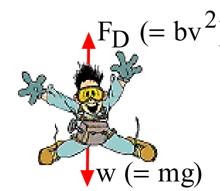
hence $g \geq a \geq 0$. When $F_D = w$ there is *no net force* so $a = 0$, and the *terminal velocity* is:

$$v_t = \sqrt{\frac{mg}{b}}.$$

NOTE: this is <u>not</u> "free fall" ... *WHY NOT ??*

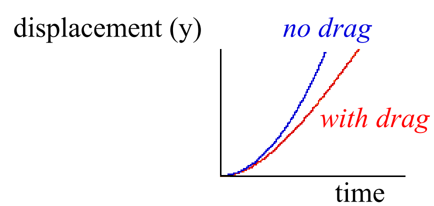






$$mg - bv^2 = ma_y.$$

$$\therefore a_y = \frac{mg - bv^2}{m}.$$



From rest:

$$(y - y_\circ) = \frac{1}{2} a_y t^2$$

$$= \frac{1}{2}\left(g - \frac{b}{m} v^2\right) t^2.$$

DISCUSSION PROBLEM [5.2]:

If a feather and a baseball are dropped from a great height - so they each reach terminal velocity - the baseball strikes the ground first. Which object experiences the greater drag force?

A: The feather.
B: The baseball.
C: They experience the same drag force.