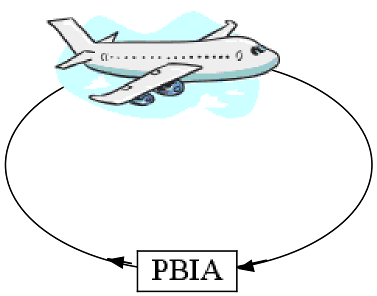
CAN A MOTHER BE YOUNGER

THAN HER DAUGHTER?

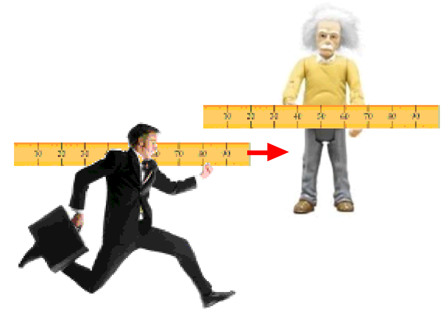
(and other curiosities of relativity)

- Everyday view (Newton and Galileo).
- Einstein's view (constancy of the speed of light).
- Two postulates of Special Relativity and their effect on space and time.
- Einstein's *famous* equation.
- General Relativity.
- Einstein's conception of gravity and *warped space*.
- Some consequences of General Relativity.

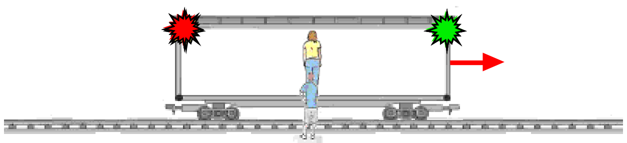
Scenario 1: Imagine two people with identical clocks, i.e., identical ticking rates. If one of them moves past the other (who is stationary), it seems entirely resonable they should *both* agree that the ticking rates are still exactly the same.



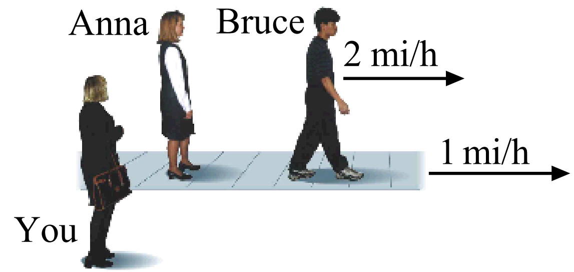
So a passenger on a round trip would expect to measure exactly the same time for the journey time using their watch as someone on the ground would measure using their watch.



Scenario 2: Imagine a stationary observer holding a 1- meter ruler, when an identical 1-meter ruler held by a runner "passes" by. It seems perfectly reasonable that both the observer and the runner will measure the length of the other person's ruler to be the exactly the same as theirs.



Scenario 3: Anna is standing in the middle of a railroad wagon, which is moving from left to right. Bruce is standing by the tracks. As Anna passes Bruce, a red firecracker at the rear of the wagon and a green firecracker at the front of the wagon are set off simultaneously. It seems reasonable that both Anna and Bruce would see the flashes at the same time.



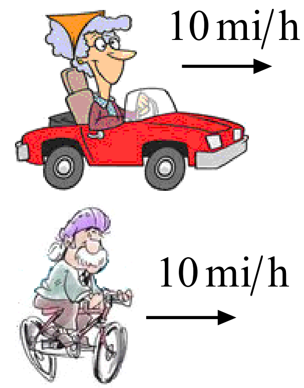
Scenario 4: You are standing and watching two people, Anna and Bruce, on a walkway. If the walkway is moving to the right with a speed of 1 mi/h, Anna, who is standing still on the walkway, will appear to you to be moving to the right at the same speed as the walkway, i.e., 1 mi/h. But, if Bruce is walking at a speed of 2 mi/h relative to the walkway, to you, he will appear to be moving with a speed of

$$2 \text{ mi/h} + 1 \text{ mi/h} = 3 \text{ mi/h},$$

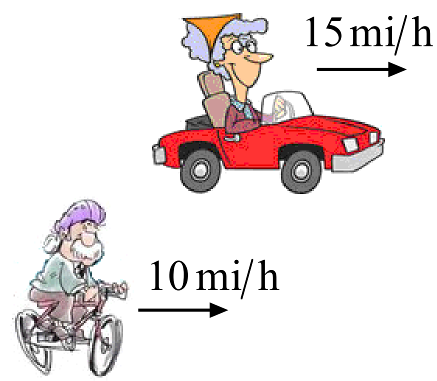
i.e., a simple addition of speeds.

It turns out that none of these conclusions is correct although the discrepancies, at everyday speeds, are so small as to be insignificant and not measurable. But, at much higher speeds, the results must be modified.

Welcome to the weird world of Relativity!

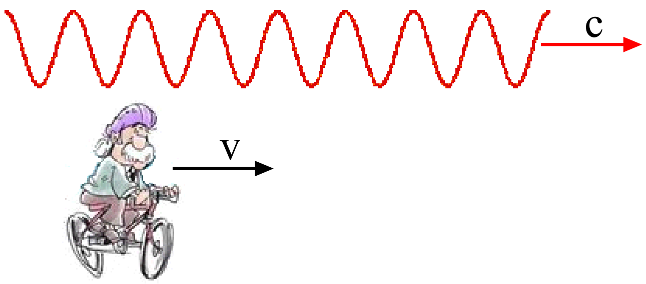


To the cyclist, the driver will appear to be stationary.



To the cyclist, the driver will be moving away from him at a speed of

$$15\text{mph} - 10\text{mph} = 5\text{mph}.$$



Now, replace the car by a beam of light. If the cyclist is traveling at a speed of v and the light beam has a speed c, would he see the light beam traveling with a speed equal to the difference,

i.e., $c - v$?

So, if the cyclist was traveling at ½ the speed of the light, would he see the light beam traveling with a speed

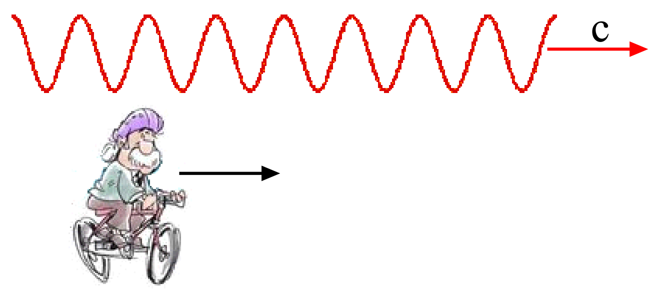
$$c - \frac{1}{2}c = \frac{1}{2}c?$$

Newton would probably have said ... YES!

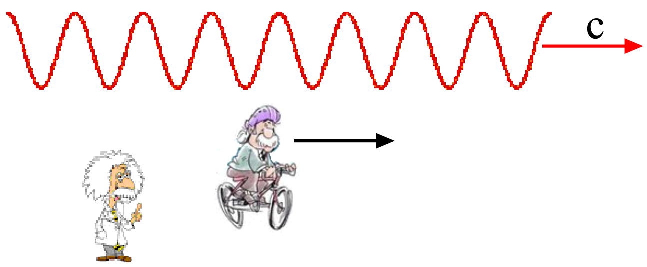
But Einstein said ...

NO !

No matter what his speed, the cyclist would always measure the same value for the speed of the light beam, i.e., the speed of light always appears to be constant, no matter what the speed of the observer even, if he's traveling close to the speed of the light beam!



How can that be?



The solution is that in order for the the cyclist to measure the speed of the light beam to be constant no matter what their speed ...

time itself must slow down when one is moving, at a rate determined by the speed!

So, the watch on the cyclist's wrist must tick *more slowly* than the watch on the wrist of someone standing by and watching what's going on. Then the light beam appears to travel just as fast for the cyclist as it does for the observer.

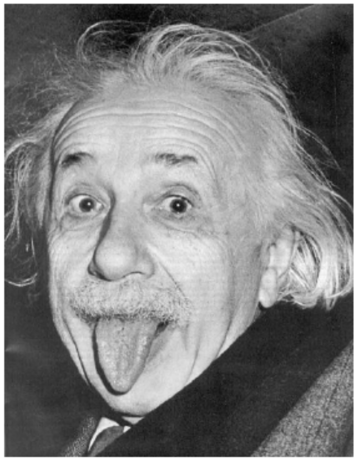
The speed of light is close to

300,000,000 m/s (186,000 mi/h).

It would take about 0.133 s for a beam of light to circle the Earth. So, in one second the beam would circle the Earth about 7.5 times.

The Zytglogge clock tower in Bern, Switzerland. Many claim it inspired Einstein to think about the relativity of time.

So, what is relativity?



Once, when he was asked to explain relativity in a few sentences, Albert Einstein replied:

> *When you are courting a nice girl, an hour seems like a second. When you sit on a red hot cinder, a second seems like an hour. That's relativity!*

- In 1905, Albert Einstein announced his *special theory of relativity.*

Special relativity

Involves a comparison of measurements made by different *observers* who are moving with constant speed relative to each other.

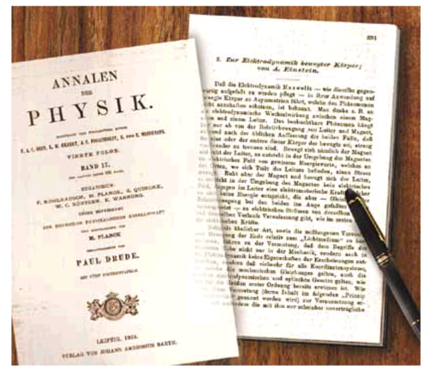
- In 1916-1920 he formulated his *general theory of relativity.*

General relativity

Involves a comparison of measurements made by different *observers* who are accelerating relative to each other and in the presence of gravity.

3. *Zur Elektrodynamik bewegter Körper;*
von A. Einstein.

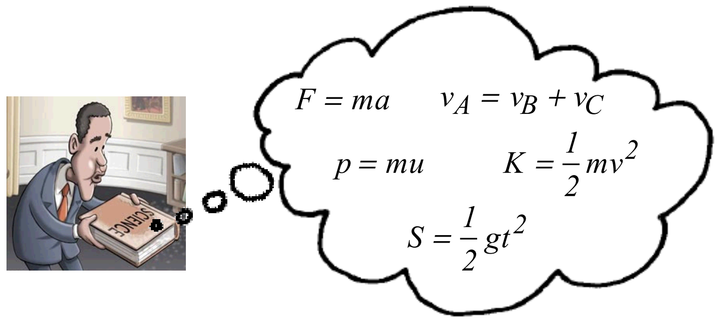
On June 30, 1905 Einstein, published a paper



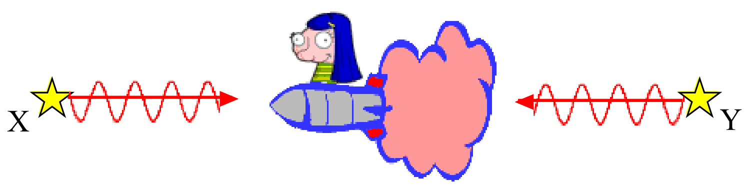
entitled "*On the Electrodynamics of Moving Bodies*", in which he introduced his special theory of relativity. The theory is based on two postulates:

- *The laws of physics are identical for all observers moving at constant velocity with respect to each other.*

- *The speed of light is constant, independent of whether the light is produced by a source that is moving or at rest.*



• **<u>Postulate 1</u>** means that some modifications are necessary to the classical laws and equations.



• **<u>Postulate 2</u>** tells us, for example, that an astronaut on a spaceship traveling from star Y to star X will measure the speed of light originating from X and Y to be the same, i.e., the speed of light is constant no matter what the speed of the person making the measurement.

<u>According to Galileo and Newton</u>:

• Distances and time were entirely separate quantities and could be treated independently of each other.

• The ideas of Galileo and Newton fit well with everyday experiences. But relativity places strict limits on some of these concepts.

According to Einstein's relativity:

• There is no such thing as absolute distances nor absolute time intervals; they are intimately interwoven.

Time intervals and distances depend on the speed of the observer making the measurements.

Scenario 1: Einstein showed that if identical clocks are in relative motion, a stationary observer finds that the moving clock ticks *more slowly* than his (stationary) clock. But, as far as the runner is concerned, his clock continues to tick at the same rate as when he was stationary.

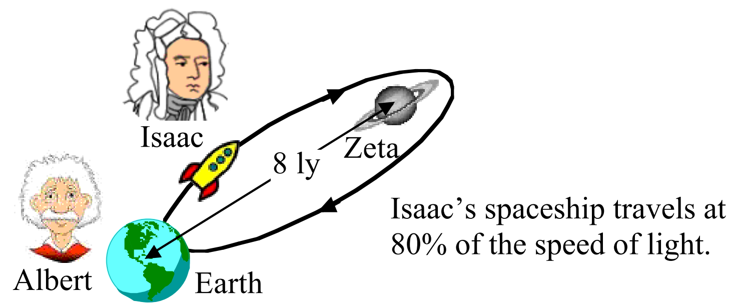
This effect is called *time dilation*.

After 60.0s have passed on a stationary clock, clocks moving at various fractions of the speed of light relative to the stationary clock will show the times indicated, because moving clocks tick more slowly than a stationary clock.

Stationary clock	60-0
50% of the speed of light	52-0
60% of the speed of light	48-0
70% of the speed of light	42-8
80% of the speed of light	36-0
90% of the speed of light	26-2
95% of the speed of light	18-7
99% of the speed of light	8-4

<u>Some interesting consequences of time dilation</u>!

• Albert and Isaac are 20 year-old twins. Isaac travels to and returns from Zeta, which is 8 light years from Earth.

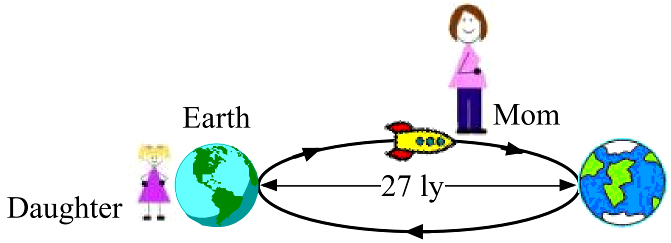


• According to Albert, Isaac takes 10y to get to Zeta and 10y to get back. Isaac's total journey time is 20y, so, when he returns, Albert will be 40 years old.

• But because of relativity, Isaac's clock (including his body clock, i.e., heart) ticks slower than Albert's. According to Isaac his total journey time is only 12y.

• So, when they get back together, Albert is 40 years old and Isaac is 32 years old . Isaac grew older more slowly!

Can a mother be younger than her daughter?

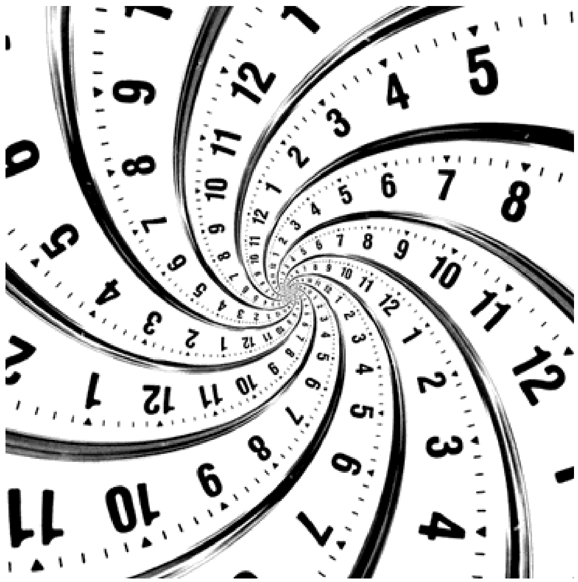


• YES! Suppose your Mom (age 30) left Earth for a distant planet 27 light years away when you were age 5.

• If she traveled at 90% of the speed of light, her journey would take 30y (Earth's time).

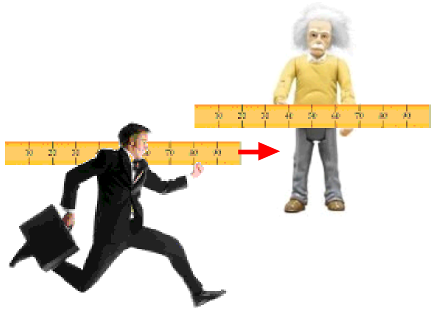
• If Mom was on the planet for 3y before returning, her time away would be $30y + 3y + 30y = 63y$. So, you would be 68 years old when Mom returns. But how old is she?

• Because of relativity, according to Mom's clock, her journey time would be $13y + 13y = 26y$. So, her total time away would then be $26y + 3y = 29y$. Consequently, Mom would only be 59 years old when she returns!



• *Is this a form of time travel?*

• *Will the mother outlive her daughter?*



Scenario 2: Einstein also showed that space, i.e., the distance between points, depends on the speed of the observer. The 1-meter ruler that is moving will appear *shorter* than the stationary 1-meter ruler. But as far as the runner is concerned the length of his ruler is still 1-meter!

This effect is called *length contraction*.

The apparent size of a 1-meter ruler moving in a direction parallel to its length at various fractions of the speed of light. Notice that even the runner is contracted in the direction of motion!

Stationary
100 cm

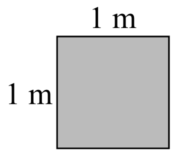
60% of the speed of light
80.0 cm

80% of the speed of light
60.0 cm

90% of the speed of light
43.6 cm

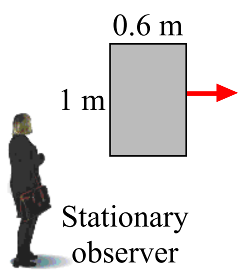
99% of the speed of light
14.1 cm

1. Some implications of length contraction.



Area = 1 sq.m

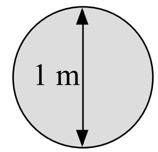
If a (1 m × 1 m) square sheet of metal passes you at 80% of the speed of light, what would it look like?



Area = 0 · 6 sq.m

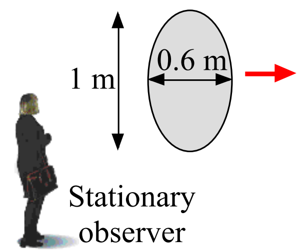
The sheet would appear rectangular, with a smaller area! Note that the contraction is only in the direction of movement.

2. Some implications of length contraction.



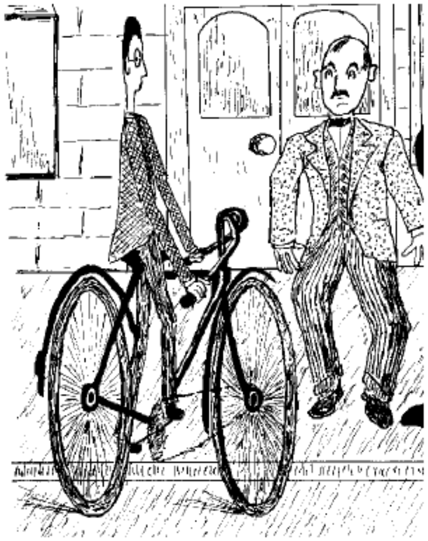
Area = 0.785 sq.m

If a 1 m diameter circular sheet of metal passes you at 80% of the speed of light, what would it look like?



Area = 0 · 471 sq.m

The sheet would appear elliptical! Note, again, the contraction occurs only parallel to the direction of motion.

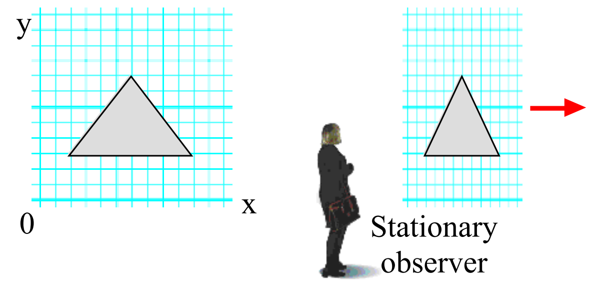


So, the wheels on a fast moving bicycle would appear elliptical to someone standing on the sidewalk. Would they see the cyclist bobbing up-and-down as they traveled along a road?

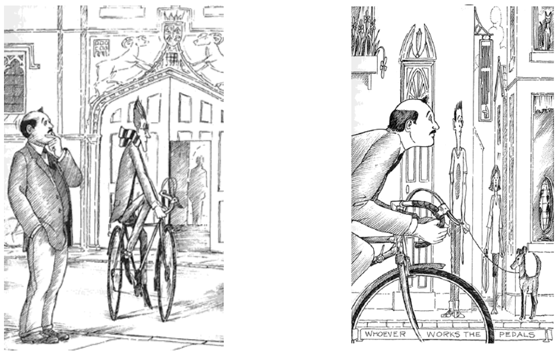
NO!

As the wheel rotates, it is <u>only</u> the horizontal dimension that is contracted and so the axle remains the same distance above the road!

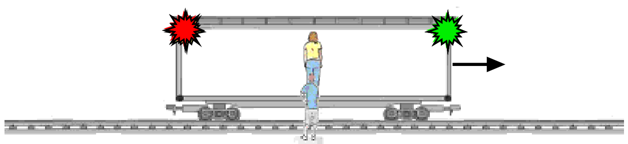
So, the Newtonian concept of *absolute space* is not correct; space (lengths) depend on the relative motion of the observer.



Space becomes *deformed* when it is in motion relative to an observer.

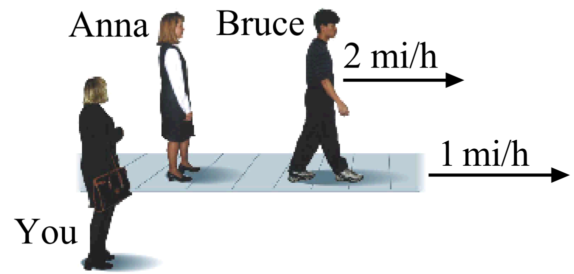


Mr. Tompkins in Wonderland.



Scenario 3: Einstein showed that events that appear simultaneous to one observer are not necessarily simultaneous to another observer moving relative to the first. So, simultaneity is not absolute; it depends on the frame of reference.

I'll demonstrate the effect in two animations.

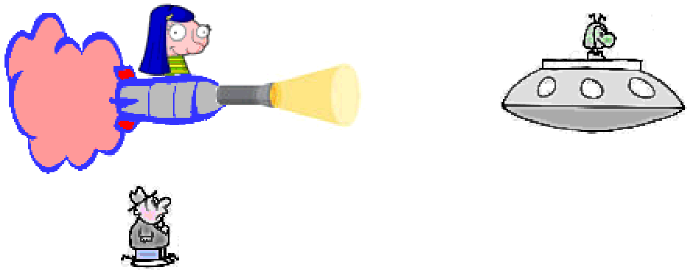


Scenario 4: Einstein showed that speeds cannot simply be added. In this example, Bruce is actually moving to the right at a speed slightly *less* than 3 mi/h in your frame of reference. At these slow speeds the difference is extremely small, i.e.,

0.00000000000000134 mi/h,

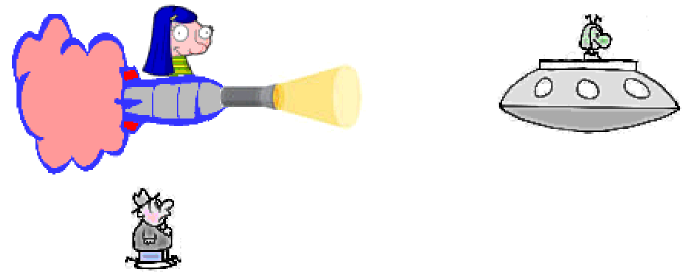
and clearly not observable!

But at higher speeds, the difference is much greater. Consider the following two examples ...

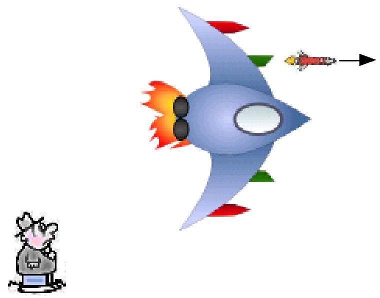


A space policewoman is chasing a speeding spaceship. To get the speeder to slow down, she switches on a light at the front of her spaceship. If the speed of her spaceship relative to the observer on Earth is 50% of the speed of light; what does the Earth-bound observer measure for the speed of the light beam?

You might be tempted to say it is the speed of light plus the speed of her spaceship, i.e., 150% the speed of light. But that is not correct.



In Einstein's special relativity, the speed of light is the same for **<u>all</u>** observers so even though the cop's spaceship is moving at 50% of the speed of light, it has no effect on the light beam as measured by her or by an observer on Earth. To the observer on the ground, therefore, the speed of the light beam is still the speed of light.

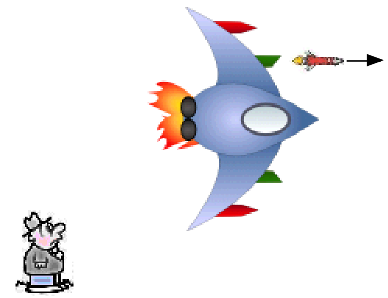


A spaceship is traveling at 50% of the speed of light, with respect to the Earth, when it fires a missile at 80% of the speed of light relative to the spaceship. What is the speed of the missile to an observer on Earth?

You might be tempted to say it is

$$50\% + 80\% = 130\% \text{ of the speed of light.}$$

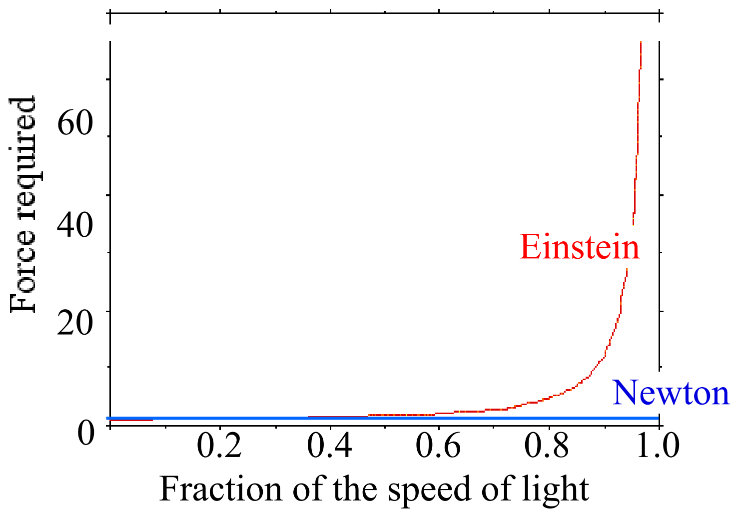
However, that is not correct.



Using Einstein's equations we find the speed of the missile measured from Earth is actually 92.9% of the speed of light.

One consequence of Einstein's equations is that no material object can travel faster than the speed of light as the force required to accelerate an object to the speed of light becomes infinite, so it is not possible to do so.

The force required to keep an object accelerating at a constant rate:



At low speeds, Newton's Laws and Einstein's relativity are essentially identical, but as the speed of the object approaches the speed of light, the force required rapidly increases and eventually becomes *infinite*.

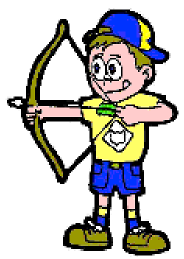
Another consequence of special relativity is that *mass and energy are equivalent* and related to each other through Einstein's famous equation

$$E = mc^2,$$

where m is the mass of an object and c is the speed of light (c = 300,000,000 m/s). So, according to Einstein, because of the *equivalence of mass and energy* an object has energy ***even when it's not moving***.

Here are a couple of examples ...

It may surprise to learn that the mass of a bow *increases*, when it is drawn ... although the mass increase is *very* small! The reason is that when



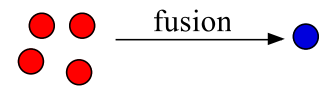
the string is pulled, the energy of the bow has increased, which through the equivalence of *mass* and *energy* manifests itself as "extra" mass. How much extra mass? Using typical figures, the increase is

$$\sim 0.0000000000000003 \text{ kg.}$$

Hardly noticeable!!

When the string is released, the extra mass is converted back into energy of the arrow. In other words, you get a lot of energy, $E = mc^2$, from a tiny amount of mass!

In the Sun the following nuclear fusion reaction occurs,



$$4H \rightarrow He + \textit{energy}$$

in which 4 hydrogen atoms are converted to a single helium atom. But the mass of 4 hydrogen atoms is actually *greater* than mass of a helium atom; the "loss" of mass appearing as "energy"

$$= mc^2,$$

where m is the loss in mass.

Mass is lost at a rate of 4.2 billion kg/s in the Sun! This may seem like a lot, but in the lifetime of the Sun (~4.5 billion years) only ~0.03% of its mass has been "lost", i.e., converted to energy.

After Einstein had explained his mass-energy equivalence equation to his wife ...

"*Hey, Albert, how about converting some of your mass into energy and clearing up this mess*?"

Adapted from the original at www.CartoonStock.com.

General Theory of Relativity

Newton's universal law of gravitation, which he introduced in 1687, proved highly successful. There were a few weaknesses, which Newton recognized, but had no answer for. For example,

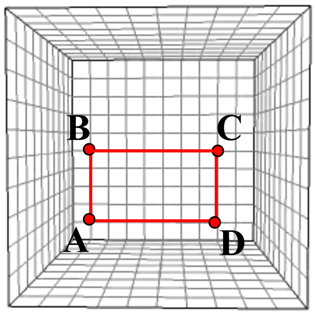
- When two objects are attracted to each other, how did each of them know the other was there?

- What was the speed of the gravitational "attraction" between objects?

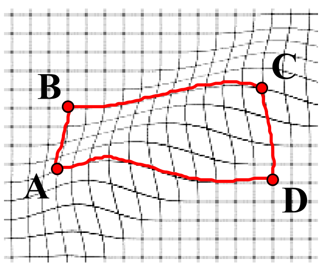
In 1907, Einstein began to think about how Newton's Law of Gravitation should be modified to fit his special theory of relativity.

What was Einstein's conception of gravity?

Space completely empty of material objects can be thought of as a uniform grid, or flat space (a). So traveling directly from one point to another can be achieved by moving in straight lines.



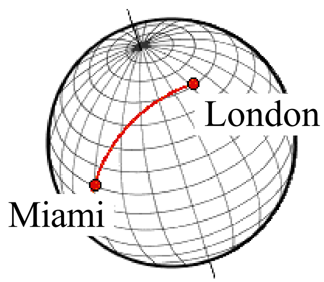
(a) Flat space



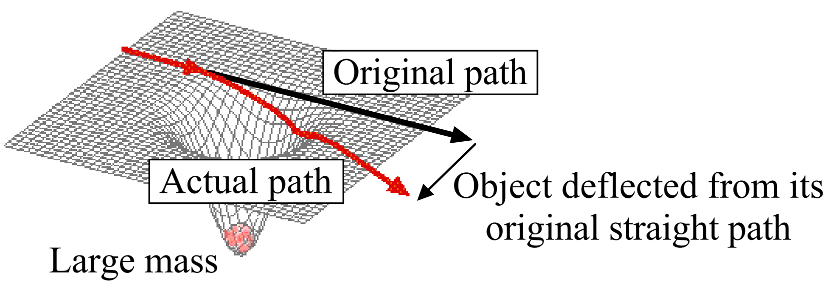
(b) Warped space

Einstein proposed that material objects "distort" space due their mass, so the gridlines become warped (b). To move in warped space, one has to follow the warped contours.

The Earth's surface is an example of curved space; to travel from Miami to London, the shortest path is necessarily curved.



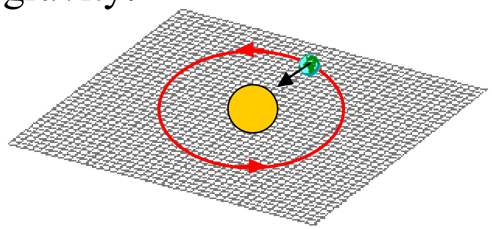
According to Einstein, the deflection of a object by a large mass occurs not because they are *attracted* to the large mass, as in the Newtonian approach, but because they travel in the curved space produced by the large mass. As a result objects are deflected *towards* the large mass giving the appearance of attraction.



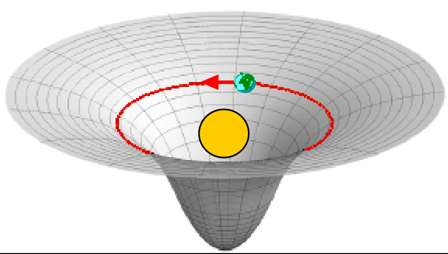
The amount of deflection depends on how close the original path is to the large mass and the amount of curvature.

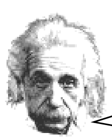
Demonstration of the effect of the warping of space.

The difference between Newton's and Einstein's concept of gravity:



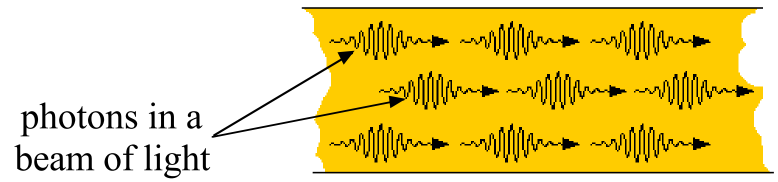
Gravity is a *force* and so it is the gravitational attraction of the Sun that keeps the Earth in its orbit.





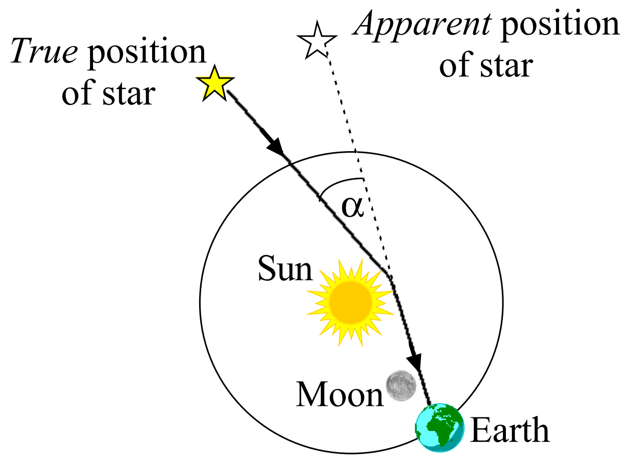
Gravity is caused by the curvature of space; the Earth in its orbit around the Sun simply follows the "*warping*".

For everyday experiences on Earth, the predictions of general relativity are essentially identical to Newton's.



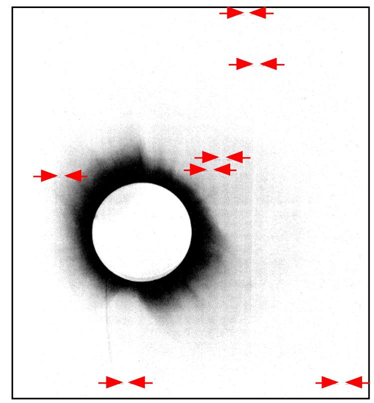
In 1905 Einstein proposed that light was composed of a stream of particles (today we call them *photons*) each with discrete energy. Since the particles have energy, by his equivalence principle, $E = mc^2$, they must have an "apparent mass". He argued that if they have mass then they should be affected by gravity so a light beam might be bent when passing close to a heavy object.

As a result, a beam of light from a star passing very near the Sun's surface would be deflected by the warping of space due to the Sun. So, the *apparent* position would be shifted from the *true* position. For a ray of light just grazing the Sun's surface, Einstein calculated the angular shift to be about 0.0005°.



So, a measurement of the deflection provides a test of the theory of general relativity.

Arthur Eddington measured the deflections of seven stars, mainly in the constellation of Taurus, during a total eclipse of the Sun on May 29, 1919.



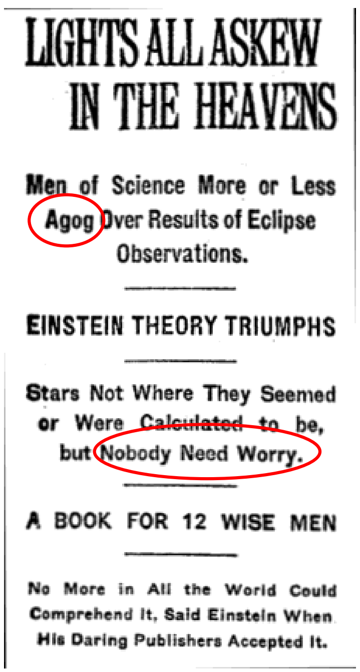
Copied from Eddington's paper published in 1920. I added the arrows for clarity.

The analysis proved extremely difficult and time consuming but, eventually, he determined the angular shift of a grazing ray to be:

$$0.00055^\circ \pm 0.00003^\circ$$

Einstein's theory had predicted 0.0005°.

This was the *first* experimental verification of the general theory and made Einstein an instant celebrity. On November 7, 1919, the ***London Times*** ran the headline "*Revolution in Science: Einstein vs Newton*".



LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

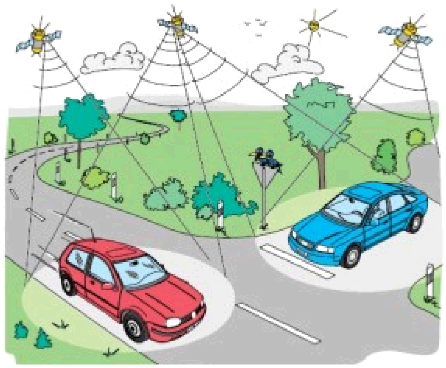
No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

New York Times headline, November 10, 1919.

Not to be outdone, on November 10, 1919, the ***New York Times*** used the word "*Agog*" in their headline, and that "*Nobody Need Worry*"!

Another consequence of general relativity is that gravity affects the passage of time. Clocks in weaker gravity run faster than identical clocks in stronger gravity.

So, if you stand on a ladder you will age faster than if you stood on the ground! The effect is extremely small. For a step 1 ft high, a life span of 80 years will be shortened by about 90-billionths of a second!



Through a combination of *time dilation* and the *gravitational effect on time*, special and general relativity predict that the clocks on the GPS satellites, orbiting at a height of 20,000 km, have a discrepancy of ~ 38 millionths of a second each day, which amounts to about 0·00000005%. Although small, corrections have to be made in order to achieve locations to with $1 \to 2$ m.

A watch on someone approaching a black hole will appear to be gradually ticking more slowly to an observer some distance away. As the person enters the black hole, their watch will appear to have stopped!

However, as far as the person falling into a black hole is concerned, their watch will appear to be ticking quite normally!

And finally, if all of this leaves you a bit confused, don't worry, take heart from the following Einstein quote ...