HOW MUCH DOES THE EARTH WEIGH?

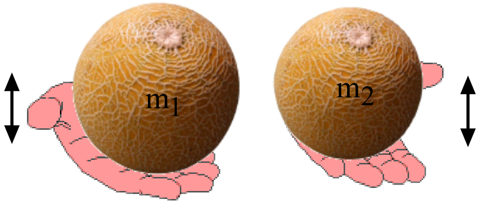
To be strictly pedantic, a better question is

WHAT IS THE MASS OF THE EARTH?

- Mass, weight and density.
- The role of Isaac Newton (*ca*. 1687).
- Early attempts to determine the density of the Earth:
    - Pierre Bouguer (1749),
    - Nevil Maskelyne (1774).
- Henry Cavendish and the Cavendish experiment (1797-98).
- Later attempts.

Some differences between mass and weight:

✷ **Mass** is related to the amount of matter or substance contained in an object. It determines how easy or difficult it is to "move" an object



✷ **Weight** is the force by which an object is attracted to the Earth (or some other body, like another planet or the Moon).

Some differences between mass and weight:

✷ **Mass** is an intrinsic property of an object as it remains the same everywhere in the universe.

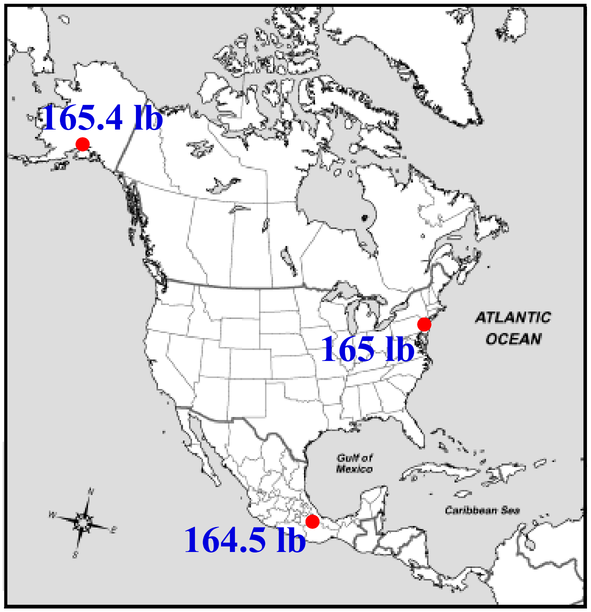
✷ **Weight** is variable depending on the gravitational force acting on the object.

If your bathroom scale reads 120 lb on Earth, it would read

25.5 lb	on the Moon
58.1 lb	on Mars
140 lb	on Saturn
364 lb	on Jupiter

Some differences between mass and weight:

If your bathroom scale reads 165 lb in New York City, it would read



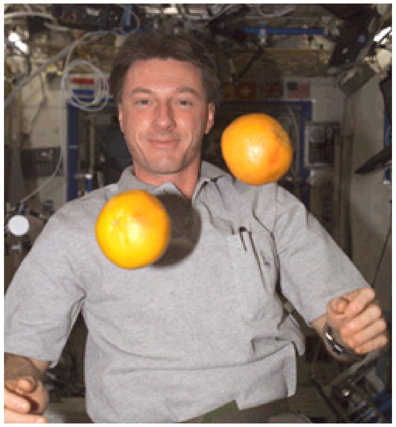
165.4 lb in Anchorage, Alaska, and

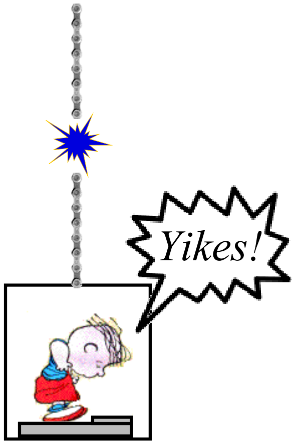
164.5 lb in Mexico City!

But your mass is the same in each city!

Some differences between mass and weight:

- **Mass** can never be zero for then the object wouldn't exist!
- **Weight** can be zero if there is no gravitational force acting on the object or it is in free-fall, for example, a grapefruit on the ISS has mass but is "weightless"; if you stand on a bathroom scale in an elevator and the chain breaks you become "weightless" but you still have mass.





Some differences between mass and weight:

✷ **Mass** can be measured using a pan or lever balance comparing the unknown mass with a known mass. The measured mass will be the same everywhere.

✷ **Weight** is measured using a spring balance. But, the actual reading would depend on location.

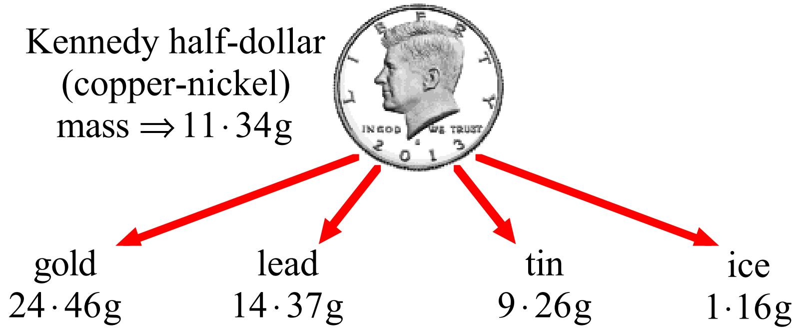
Another quantity we will come across is:

✷ **Density**. Is physical property of matter, as each element and compound has a unique density.

$$\text{Density} = \frac{\text{Mass of an object}}{\text{Volume of the object}}.$$

$$\therefore \text{ Mass of an object}$$

$$= \text{density} \times \text{volume of the object}.$$



Often, the density of a substance is compared with the density of water (the relative density).

Examples of densities relative to water:

Substance	Relative density
Cork	~0.25
Pine	~0.5
Hickory	0.83
Tin	7.31
Iron	7.87
Nickel	8.91
Copper	8.96
Silver	10.50
Lead	11.35
Uranium	19.10
Gold	19.32
Platinum	21.45
Iridium	22.56
Osmium	22.59

Clearly, we cannot measure the mass of the Earth directly. Even though the volume of the Earth was known, its density could only be guessed at as the density of soil is about $1 \rightarrow 2\times$ that of water and typically rocks are $2 \rightarrow 3\times$ times that of water. So, what to do?

Isaac Newton provided the clue in his Law of Gravitation:

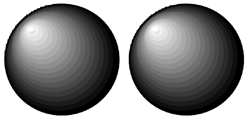
> "*any two bodies in the universe* *__attract__ each other with a* *__force__ that is directly* *__proportional__ to the product of their masses and inversely proportional to the square of the distance between them.*"
>
> Principia Mathematica, Book III
> Props 6,7.

Using his Law of Gravitation and the astronomical data known at the time, Newton was able to calculate the relationship between the densities of the Earth and planets with moons, taking the relative density of the Earth = 1.00.

	Earth	Saturn	Jupiter	Sun
Newton	1.00	0.168	0.236	0.25
NASA	1.00	0.125	0.240	0.255

Densities of the Sun and planets calculated by Newton in his *Principia* (3rd edition, 1726) and the latest values from NASA.

He had no knowledge of the <u>actual</u> density of the Earth. However, he did think it was between 5 and 6 times greater than that of water.



Newton considered the possibility of measuring the attraction between two objects directly. He imagined two solid spheres, each 1 foot in diameter, made of the same material as the Earth. He stated that if they were spaced $\frac{1}{4}''$ apart, they would not

> "*even in spaces void of resistance come together by the force of their mutual attraction in less than a month's time.*"

He went on to say,

> "*Nay, whole mountains will not be sufficient to produce a sensible* [measurable] *effect.*"

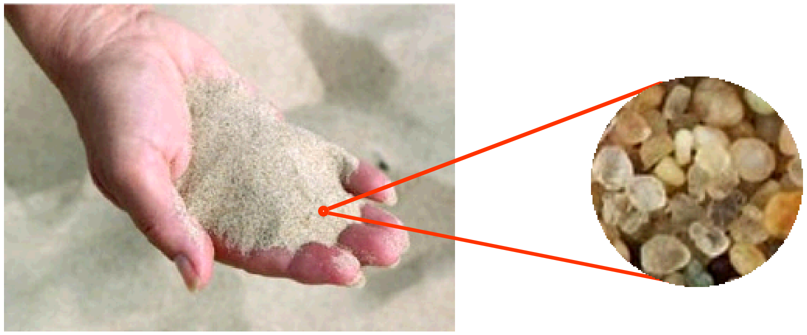
Newton had made an error leading to an enormous underestimate of the attractive force. In fact, the two solid spheres would come into contact in about $5\frac{1}{2}$ minutes instead of a month!

Had he realized his mistakes, he would not have been so quick to dismiss the possibility of measuring the attraction. Fortunately, not everyone agreed with him at the time!

But, he had recognized that there were two possible methods,

- using the deflection of a plumb-line by a mountain, and
- by measuring the force between two spheres directly.

The gravitational force of attraction between two people, each of 150 pounds sitting 18″ apart, is approximately equivalent to the weight of a single grain of sand.

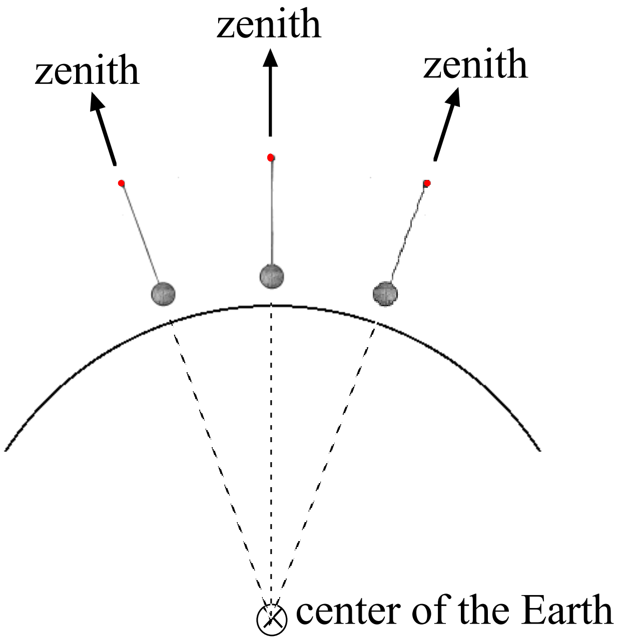


Which means the force is about

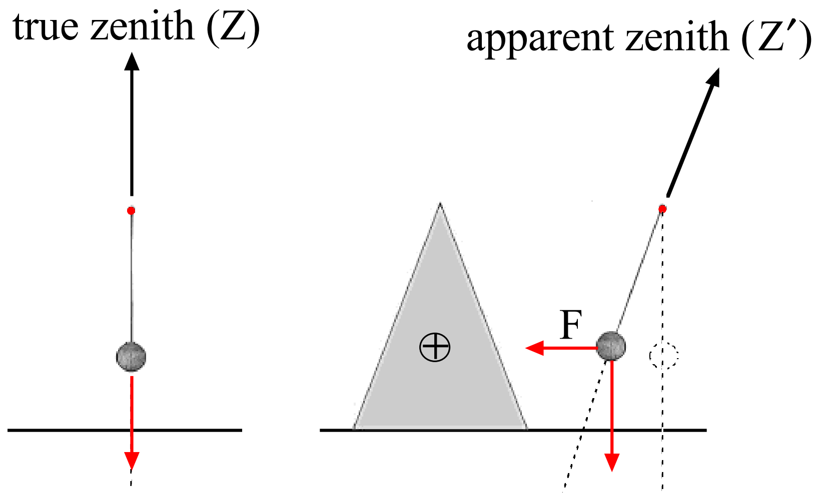
0.000000002 times,

or two-billionths, the weight of each person!

One consequence of Newton's Law:

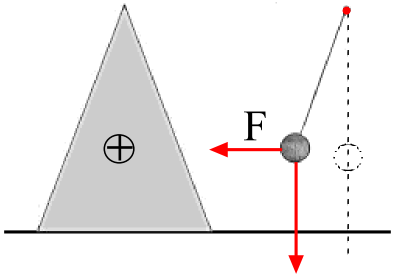


A line drawn through freely hanging, stationary pendulum, or plumb-line, on a smooth, spherical Earth would *always* pass through the center of the Earth and point upwards towards the zenith.



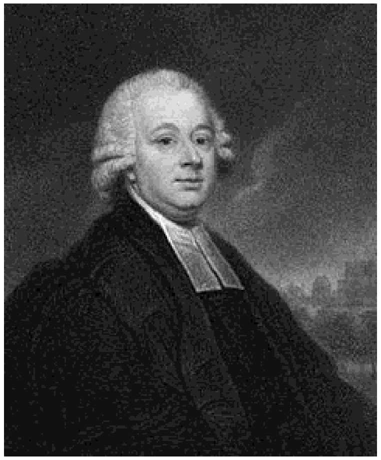
However, a line drawn through a freely hanging plumb-line near a massive object, like a mountain, will not pass through the center of the Earth. The gravitational force of attraction (F) due to the presence of the mountain will deflect the plumb-line through a small angle, so it will not point towards the true zenith. In principle, by measuring the angle, the mass of the Earth can be determined in terms of the mass of the mountain.

In 1749, Pierre Bouguer (1698-1758) performed such an experiment on Chimborazo in Ecuador. He found that a plumb-line was deflected by about 0.002°, proving Newton's proposition.



But Chimborazo is an extinct volcano made of light volcanic matter and with voids, making accurate estimates of the mass of the mountain exceedingly difficult. So, Bouguer had little confidence in his result. He recommended that more suitable mountains be found in France or England for repeat experiments.

Schiehallion (3,550 ft), a mountain located in central Scotland near Loch Rannoch, was selected as a good candidate for deflection experiments. It is almost conical in shape making a determination of its volume a relatively straightforward task.



Dr. Nevil Maskelyne FRS (1732-1811)

In 1774, Nevil Maskelyne began making measurements of the deflection of a plumb-line on the north and south sides of Schiehallion. He showed that a plumb-line was indeed deflected (by about 0.0032°) but it took a further two years for the mountain to be surveyed to provide its volume and mass.

Charles Hutton (1737-1823) depicted on a bronze medal in 1821 (National Portrait Gallery, London).

A detailed survey of Schiehallion was carried out by Charles Hutton. He estimated its density was about 2.5 times that of water, which meant the Earth was about 4.5 times as dense as water. In 1821, following a more detailed study, he revised his result upwards to 4.95. Using this latter result, the mass of the Earth would be,

5,230,000,000,000,000,000,000,000 kg,

i.e., 5.23 septillion kilograms!

Sketch of Henry Cavendish (1731-1810), dressed in a favorite, old-fashioned frock coat and three-cornered hat. Cavendish refused to sit for a portrait; this sketch was drawn surreptitiously by William Alexander at a Royal Society dinner (*ca* 1800). It is held in the British Museum.

- In 1766, he discovered hydrogen gas.

- He spent several years studying electrical phenomena but published only two papers on his researches, in 1772 and 1776.

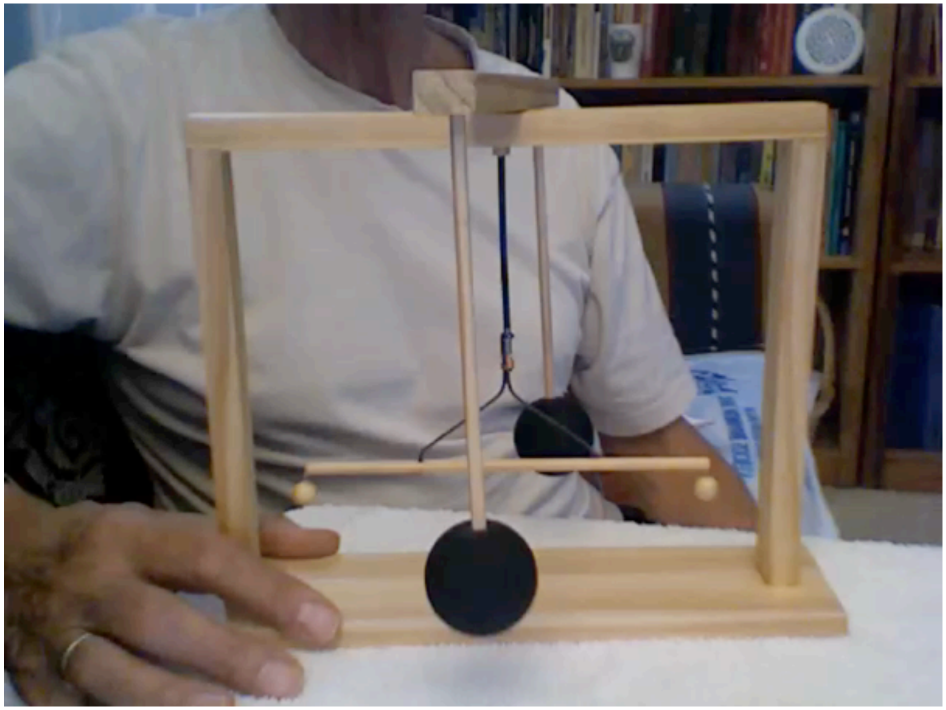
- In 1781, he showed that hydrogen and oxygen when burned together produce water.

- In 1783, he showed that the weight of water produced was equal to the weight of the two gases.

- In 1795, he discovered nitric acid by exposing mixtures of air and hydrogen to electrical discharges. He also discovered the chemical formula for nitric acid.

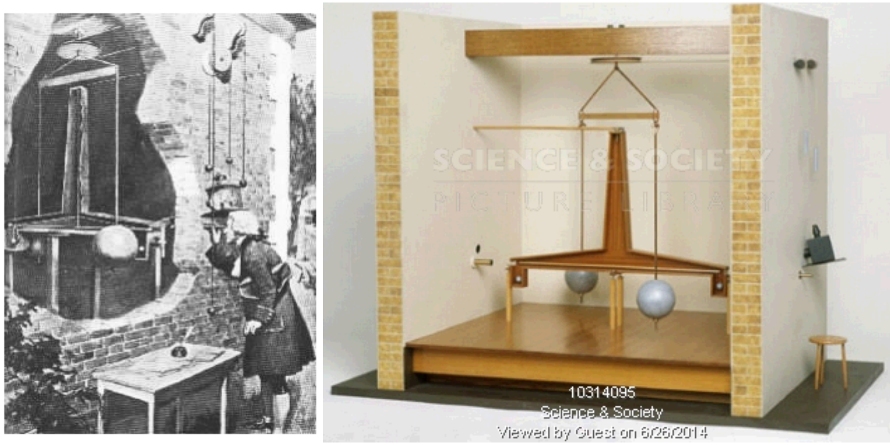
Extract from his paper read to the Royal Society in 1798:

“*Many years ago, the late Rev. John Michell, of this Society, contrived a method a determining the density of the Earth, by rendering sensible* [measurable] *the attraction of small quantities of matter; but as he was engaged in other pursuits, he did not complete the apparatus until a short time before his death, and did not live to make any experiments with it. After his death, the apparatus came to the Rev. Francis John Hyde Wollaston, Jacksonian Professor at Cambridge, who, not having conveniences for making experiments with it, in the manner he could wish, was so good as to give it to me.*”



Model of the apparatus used by Henry Cavendish to measure the relative density of the Earth. He started his experiments in 1797.

Cavendish's house on the southside of Clapham Common, in southwest London. He lived there from 1782-1810 and it is where he carried out his historic experiment to determine the density of the Earth. The house was demolished in 1905.



Cavendish carried out his experiments inside a $10' \times 10'$ brick outbuilding at his house in Clapham, London; he operated the equipment from the outside. He observed the deflection of the torsion pendulum using telescopes at each end of the building and moved the large masses into position by a system of pulleys.

He made a series of 17 separate experiments altogether, between August 5th, 1797 and May 23rd, 1798, comprising 29 measurements of the density. He concluded the density of the Earth was

$$5.448 \pm 0.033$$

times that of water.

Assuming the Earth is a sphere with an average radius of 6371 km, then its mass is

$$5{,}901{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ kg},$$

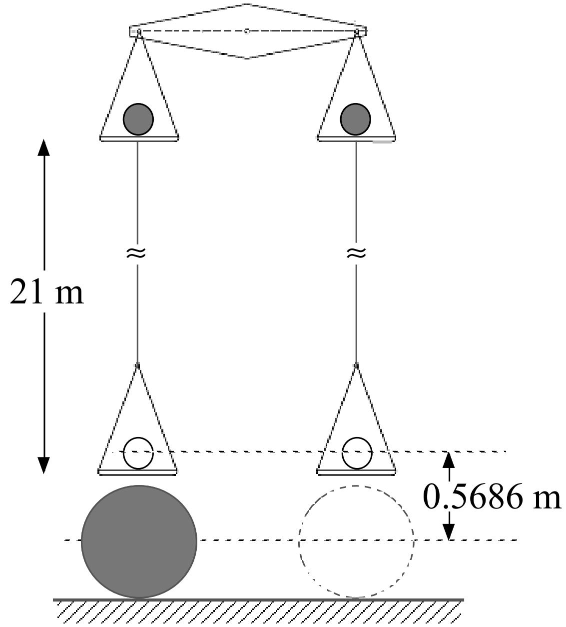
which is within 1.2% of today's accepted value.

His experiment, which involved the elimination of many spurious effects, has been heralded as one of the greatest experiments in the history of science.

Above: the ‘old’ Cavendish Laboratory (1874).

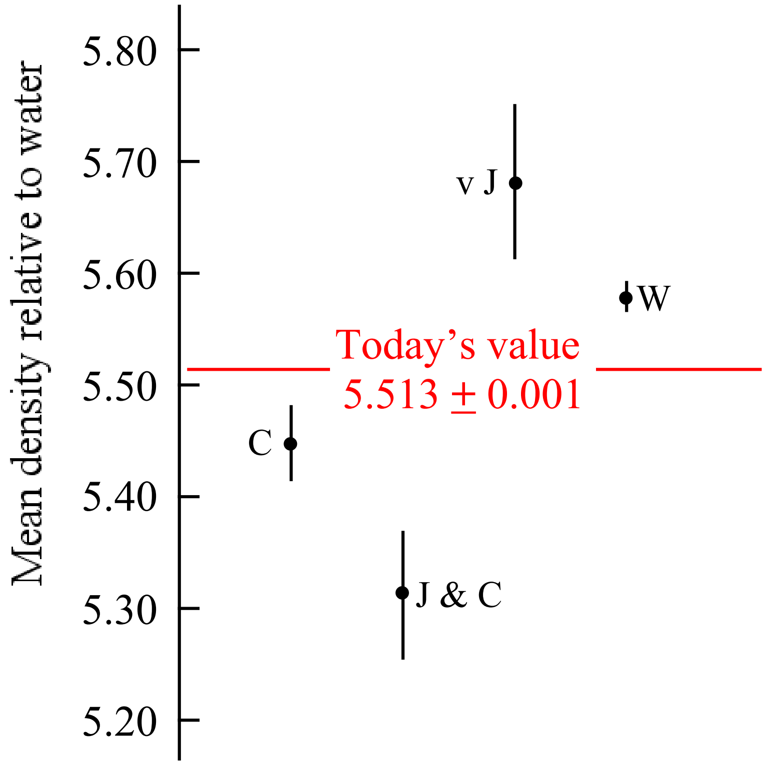
Below: the ‘new’ Cavendish Laboratory (1974).

In 1856, Colonel Sir Henry James and Captain A.R. Clarke carried out a repeat of Maskelyne and Hutton’s plumb-line experiment but this time using Arthur’s Seat, a prominent hill overlooking Edinburgh in Scotland. They assumed the average density of the hill was 2.75 times greater than that of water and obtained a value of 5.316 ± 0.054 for the density of the Earth compared with that of water $(= 1.000)$.



In 1879, Phillipp von Jolly carried out a series of experiments using a modified common beam balance and four glass globes identical in size - two containing mercury and two evacuated - and a large lead ball, weighing 5,772 kg (about 6 tons). He found the average density of the Earth was 5.692 greater than that of water.

Model to the apparatus used by Johann Wilsing to measure the relative density of the Earth. His experiments commenced in 1887.



Key:

C - Cavendish (1798)
J & C - James and Clarke (1856)
v J - von Jolly (1881)
W - Wilsing (1887)

A comparison of the densities of the planets (relative to water) in the solar system.

Planet	Relative density
Mercury	5.42
Venus	5.25
Earth	5.51
Mars	3.94
Jupiter	1.31
Saturn	0.70
Uranus	1.29
Neptune	1.64

Today, the best estimate of the Earth's mass is

5,972,000,000,000,000,000,000,000 kg,

i.e., 5,972 septillion kg.