An excursion through mathematics and its history (and some trivia)

MATH DAY 2014—TEAM COMPETITION

Sponsored by

A quick review of the rules

- History (or trivia) questions alternate with math questions
- Math questions are numbered by MQ1, MQ1, etc. History questions by HQ1, HQ2, etc.
- Math answers should be written on the appropriate sheet of the math answers booklet.
- History questions are multiple choice, answered using the clicker.
- Math questions are worth the number of points shown on the screen when the runner gets your answer sheet. That equals the number of minutes left to answer the question.
- Have one team member control the clicker, another one the math answers booklet

Rules--Continued

- All history/trivia questions are worth 1 point.
- The team with the highest math score is considered first. Next comes the team with the highest overall score, from a school different from the school of the winning math team. Finally, the team with the highest history score from the remaining schools.

HQ0-Warm Up, no points

- Which of the following was written by Descartes.

A. I think I want some jam.
B. I think my name is Sam.
C. I think, therefore I am.
D. I am, therefore I think.
E. Here I am, pour me a drink.

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Time's Up!

THE CHALLENGE BEGINS

VERY IMPORTANT!

Put away all electronic devices; including calculators.
Mechanical devices invented more than a hundred years ago, are OK.

HQ1. Miletus

Miletus is the birth place of a semi-legendary first Greek mathematician. His name was

A. Thales.
B. Anaximander.
C. Pythagoras.
D. Democritus.
E. Mathematicus.

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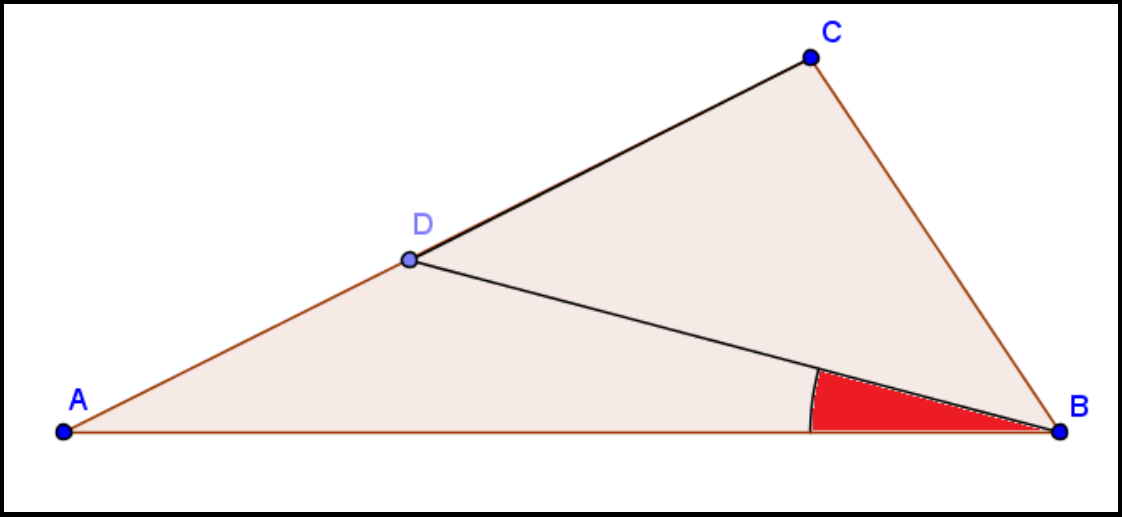
Times Up!

MQ1.A Triangular Question



D is a point on side AC of triangle ABC.

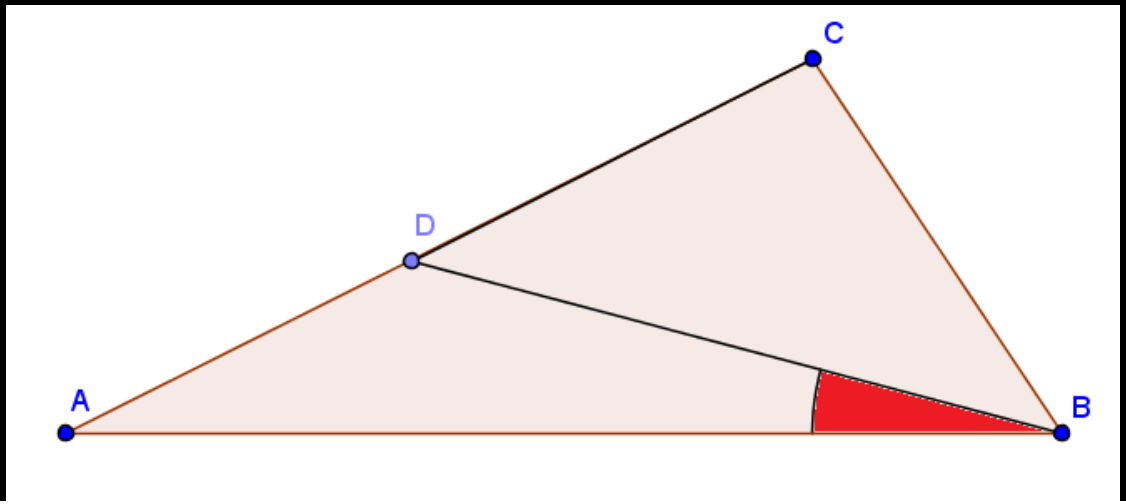
$|BD| = |BC|$

The angle of the triangle at B minus the angle at $A = 30°$.

That is, in symbols: $(\angle ABC) - (\angle BAC) = 30°$.

What is $\angle ABD$ in degrees?

MQ1. A Triangular Question



$$\begin{aligned}
\angle BDC &= \angle DBC \quad \text{(isosceles triangle)},\\
&= \angle ABC - \angle ABD, \quad \text{so } \angle BDC = \angle ABC - \angle ABD,
\end{aligned}$$

$$\angle BAC + \angle ABD + \angle BDA = 180^\circ \quad \text{(sum of angles of a triangle)},$$

$$\angle BAC + \angle ABD = 180^\circ - \angle BDA = \angle BDC = \angle ABC - \angle ABD,$$

$$2\angle ABD = \angle ABC - \angle BAC = 30^\circ.$$

$$\boxed{\angle ABD = 15^\circ.}$$

HQ2. The Sand Reckoner

Archimedes wrote a short treatise by this name for the following purpose:

A. To show sand has to be reckoned with.
B. To estimate the number of grains of sand in the Sahara.
C. To compute areas.
D. To prove that there is no such thing as a largest number.
E. To find the volume of cones and spheres.

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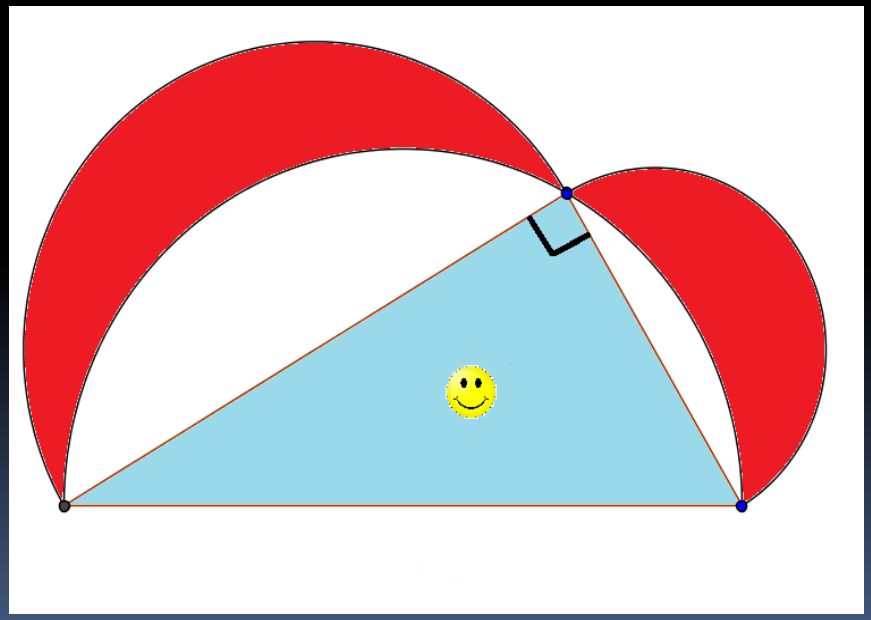
Time's Up!

MQ2. Crescent Creations

- A right triangle has area 18 (square units), semicircles are drawn on its sides as in the picture below. Find the joint area of the crescents these semicircles form, in red in the picture below. If necessary use 3.14 as the value of π.



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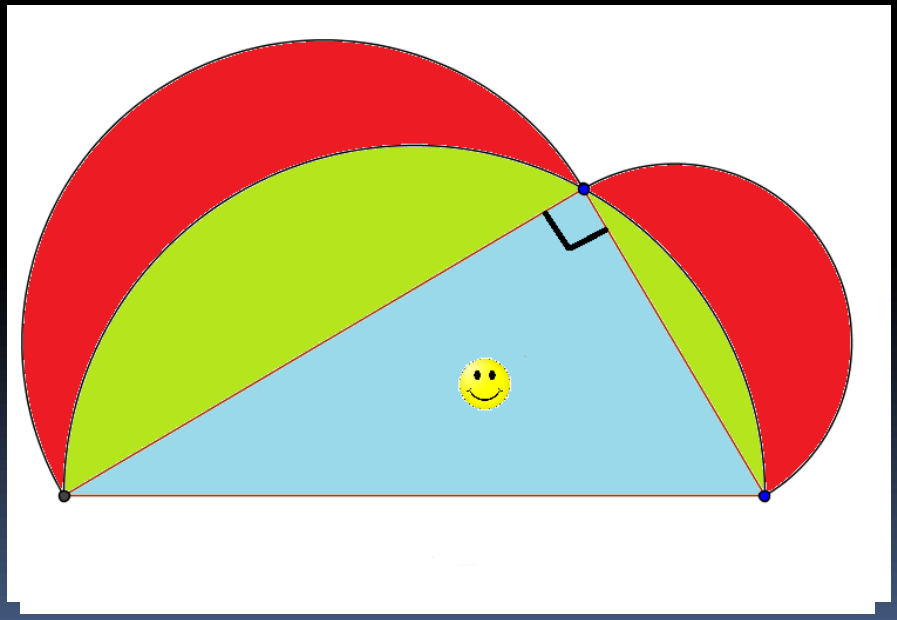


Answer = 18

Red+green = green + triangle,

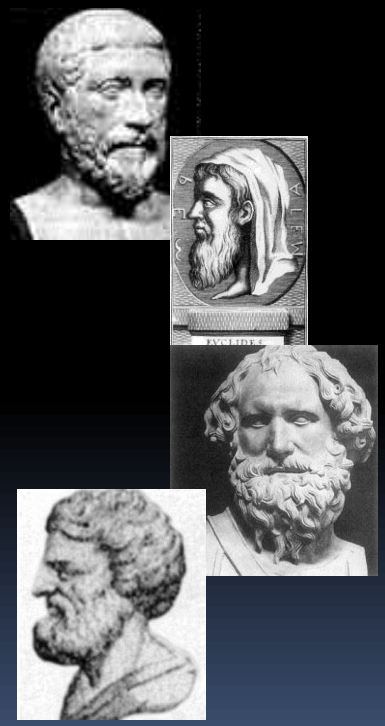
so

Red=triangle

HQ3. Conical Conceits

- His treatise on conics was one of the most important and remarkable books of antiquity. He is

A. Pythagoras of Samos
B. Euclid of Alexandria
C. Freddy of Boca Raton
D. Archimedes of Syracuse
E. Apollonius of Perga



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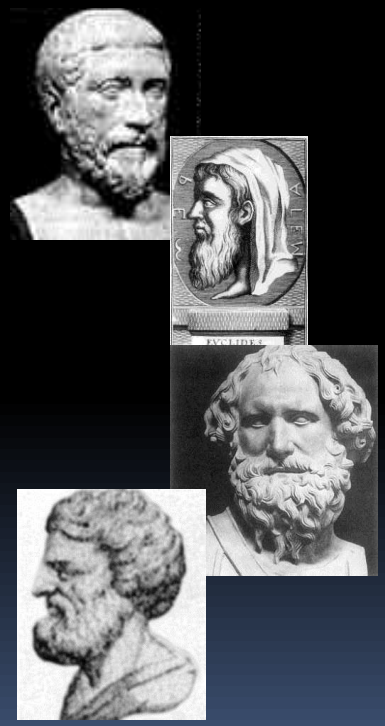
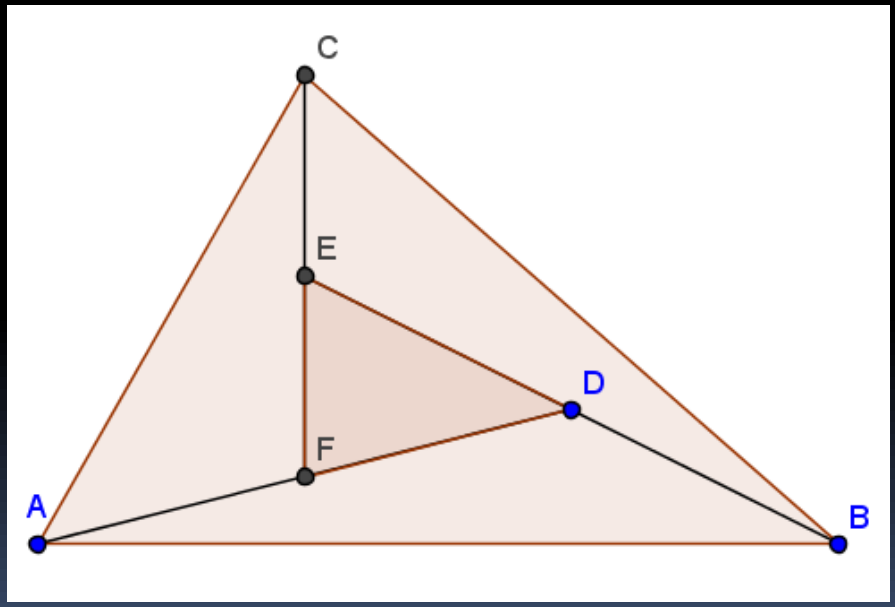


MQ3-Triangles

- In triangle ABC segments were drawn in such a way that D, E,F are the midpoints of the segments EB, CF, AD, respectively. If the area of triangle DEF is 1, find the area of triangle ABC.



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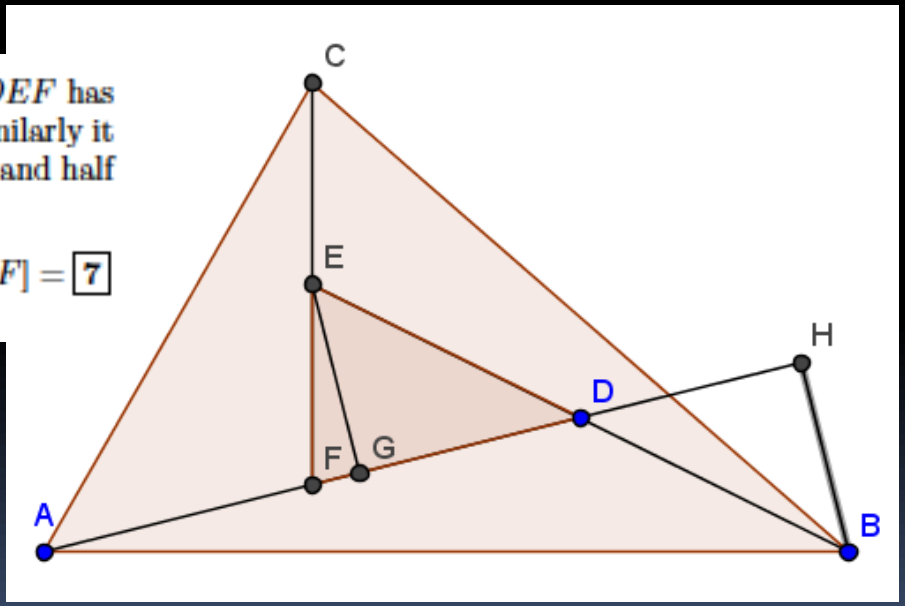
The picture shows that triangle DEF has half the area of triangle ADB. Similarly it has half the area of triangle AFC and half the area of BEC. Thus

$$[ABC] = 3 \times (2 \times [DEF]) + [DEF] = \boxed{7}$$



HQ4. Pure Trivia

"Keeler's Theorem" is a theorem by one of the writers of a popular TV show. The writer has a Ph.D. in mathematics and he came up with the theorem to solve a problem in the show. The show was

A. Futurama
B. Numb3rs
C. Family Man
D. The Big Bang Theory
E. Elementary

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MQ4. Number Friends

A pair of numbers is *amicable (friendly)* if each is the sum of the proper divisors of the other one.
Example: 284 and 220.

Divisors of 284: 1, 2, 4, 71, 142,

1 + 2 + 4 + 71 + 142 = 220.

Divisors of 220 = 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110,

1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 +44 + 55 + 110 = 284.

1184 has a friend. Find it!

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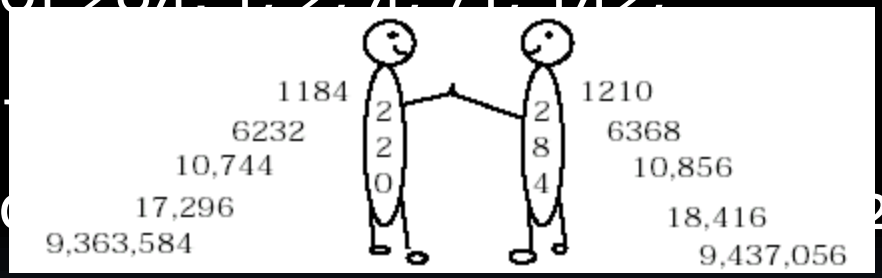

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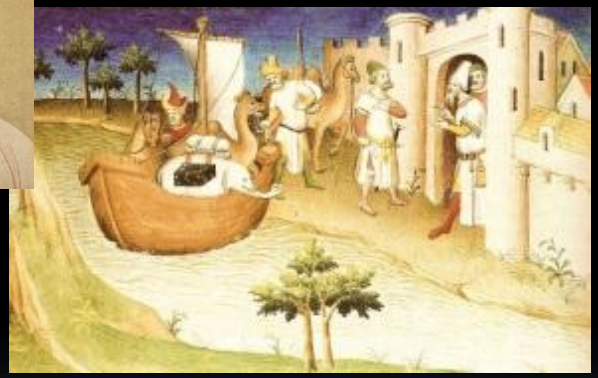
Answer 1210

HQ5. A Brief Visit to China

When Marco Polo visited China from 1275 to 1292, the emperor of China was

A. Genghis Khan
B. Kublai Khan
C. Xuantong
D. Yuan Shikai
E. Ming the Merciless



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In Xanadu did Kublai Khan
A stately pleasure dome decree
Where Alph, the sacred river, ran
Through caverns measureless to man
Down to a sunless sea
(Samuel Taylor Coleridge)

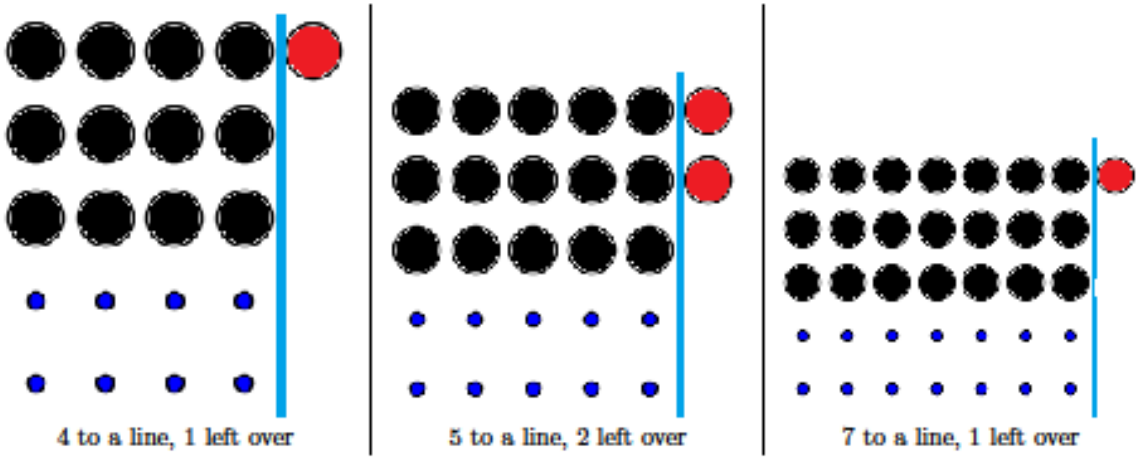
MQ5. Chinese Remainders

The marchers in the annual Math Department parade lined up.

When 4 abreast, there was 1 odd person.

When 5 abreast, there were 2 left over.

When they tried 7 to a line, there was again 1 left over.



What is the smallest number of members this department can have?

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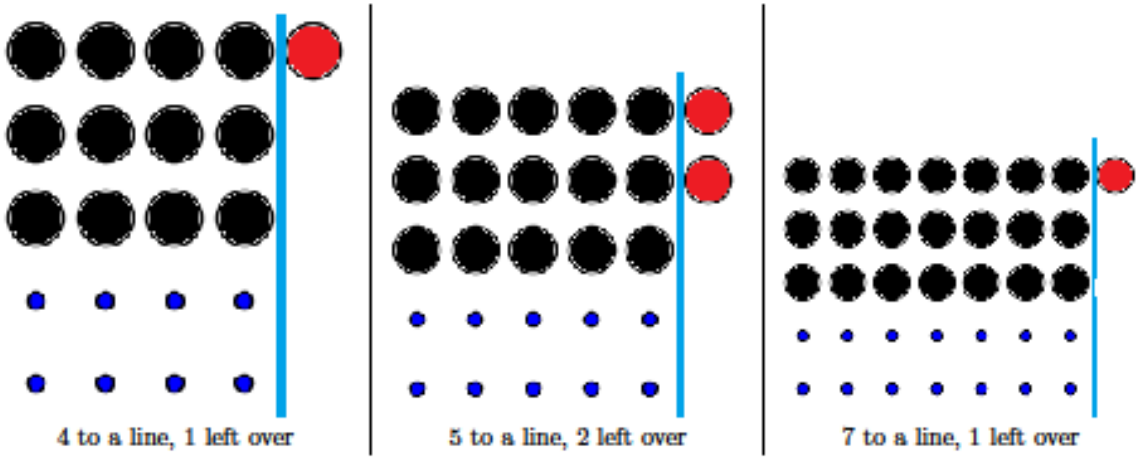

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Answer: 57

HQ6. Fibonacci's Feats

Fibonacci's most famous book (though not his best) is *Liber Abaci*, published in 1202. One of his main purposes in writing this book was:

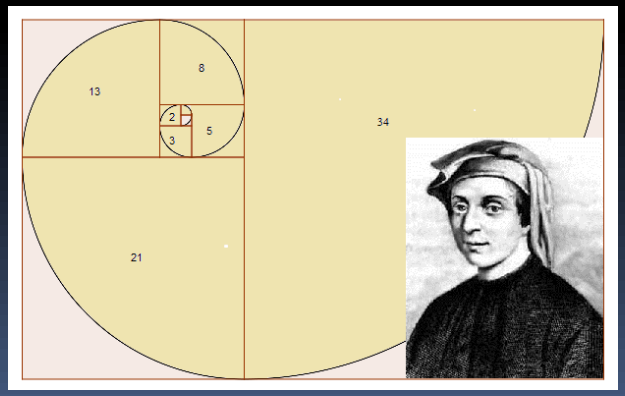
A. Introduce the Indian decimal notation to Europe.

B. Teach people to use the abacus.

C. Show how the sequence he invented could be used.

D. Popularize the quadratic formula.

E. Teach people how to count rabbits.



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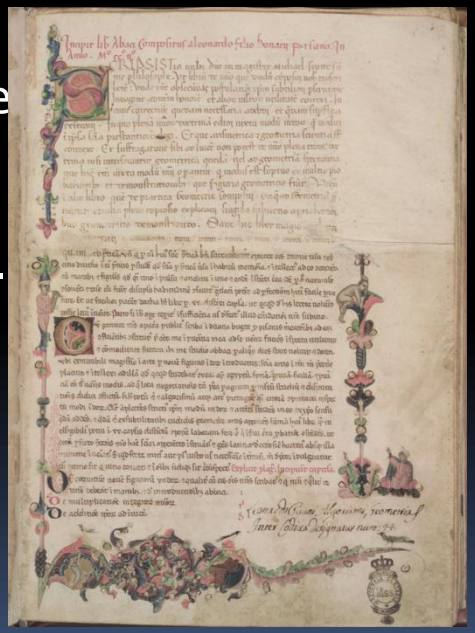


The nine Indian figures are

9 8 7 6 5 4 3 2 1

With these nine figures and 0 any number can be written

MQ6. Finding squares

- Certain two digit numbers have the property that when added to the two digit number with the same digits in reverse order, the result is a perfect square. How many such numbers are there?
- For example, $29+92 = 121 = 11^2$
- So there are at least two: 29, 92.

MQ6. Finding squares

- Certain two digit numbers have the property that when added to the two digit number with the same digits in reverse order, the result is a perfect square. How many such numbers are there?

If a, b are the digits of such a number, we are saying that $(10a + b) + (10b + a)$ is a perfect square; i.e., $11(a + b)$ is a perfect square. The only way this can happen is if 11 divides $a + b$. Since $1 \leq a, b \leq 9$, this means that $a + b = 11$. The numbers in question are

$$29,\ 38,\ 47,\ 56,\ 65,\ 74,\ 83,\ 92,$$

eight in all.

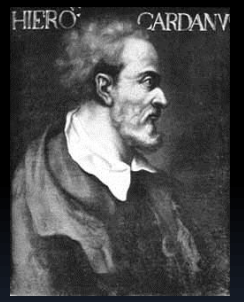
The answer is 8.

HQ7. Bologna Fights

- It was the big fight of the early 1500's. Tartaglia said he had discovered it, but it was to be a secret. Cardano, swearing never to reveal the secret, got Tartaglia to tell him. Then Cardano revealed the secret to the world. Tartaglia sued… It was a big, exciting mess. It was all about:

A. Squaring the circle.
B. Trisecting angles.
C. A method for computing volumes.
D. Computing square roots.
E. Solving cubic equations.

Tartaglia



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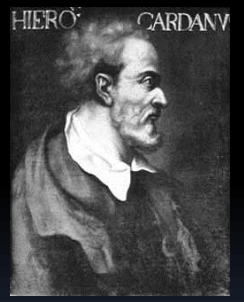


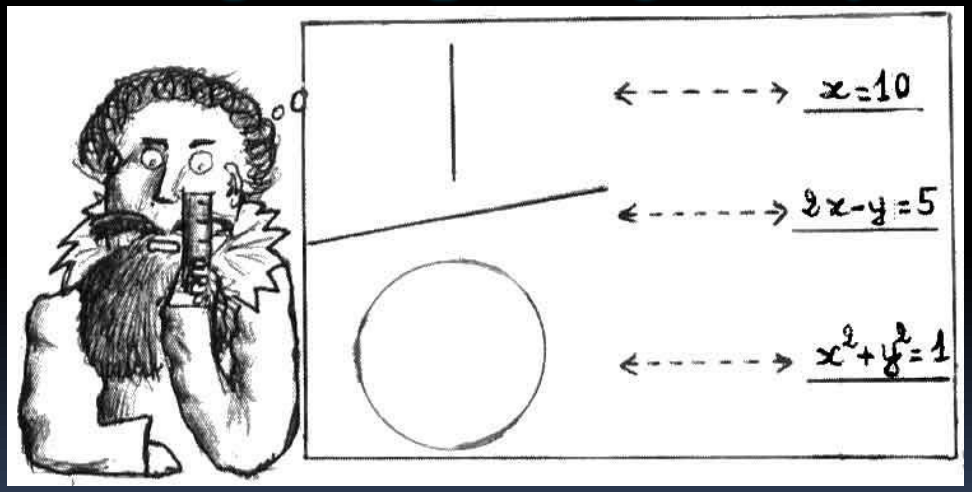
Cardano

Time's Up!

MQ7. The Equation Experience

The equation $x^4 - 5x^3 - 5x^2 + 27x + 18 = 0$ has 4 distinct real roots x_1, x_2, x_3, x_4. Compute

$$x_1^2 + x_2^2 + x_3^2 + x_4^2$$



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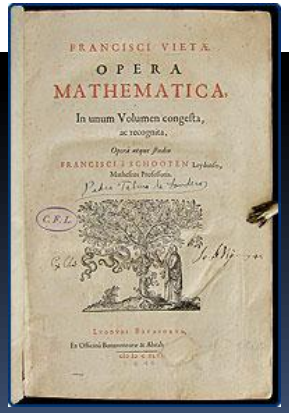
By Viète's equations,

$$\begin{aligned} x_1^2 + x_2^2 + x_3^2 + x_4^2 &= (x_1 + x_2 + x_3 + x_4)^2 - 2(x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4) \\ &= (-(-5))^2 - 2(-5) = 35. \end{aligned}$$



The answer is 35

HQ8. Sophie's Choice

She was born in the year of the American Revolution, in far away France. She wrote papers under the pen name of Monsieur Le Blanc, being afraid that in those unenlightened days (have we progressed enough?) a female mathematician would not be taken seriously. She earned the admiration of some of the top mathematicians of her day. She was

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B. Sophie Germain
C. Sophie Gregoire
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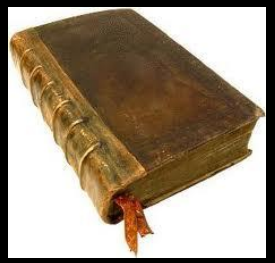

- To number the pages of a thick book a printer used 2013 digits. How many pages does the volume have?

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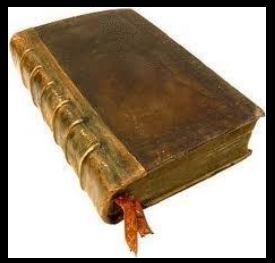

- To number the pages of a thick book a printer used 2013 digits. How many pages does the volume have?

It takes 9 digits for the first 9 pages. Then come two digit numbers, 90 two digit numbers up to page 99. That's $9+2\times 90 = 189$ digits to get to page 99. Now $2013-189 = 1824$; 1824 more digits were used. We see that 1824 divide by 3 is 608, so it will take another 608 pages to exhaust 2013 digits. $99+608 = 707$, so the answer is

707

HQ9. The Calculus Wars

- He and Newton share the glory (and bitterly fought about who should get the credit) of being the inventors of Calculus. He was a linguist, a logician, a diplomat, as well as a mathematician. Voltaire made fun of his rosy outlook on life by caricaturing him as Dr. Pangloss in Candide. He was

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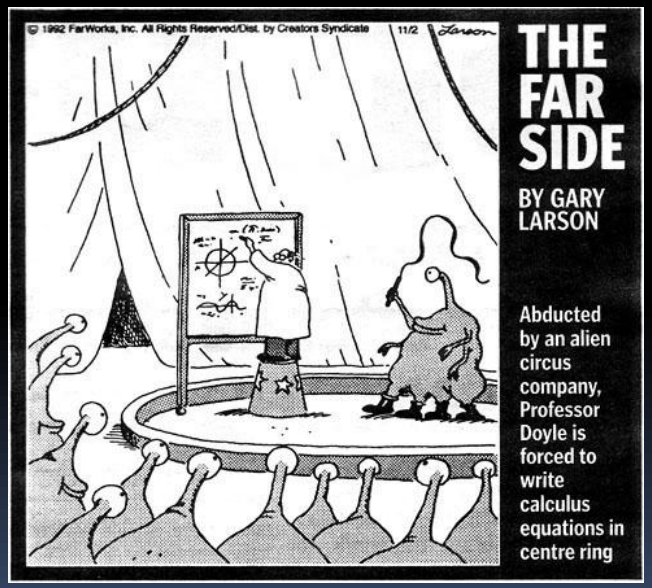

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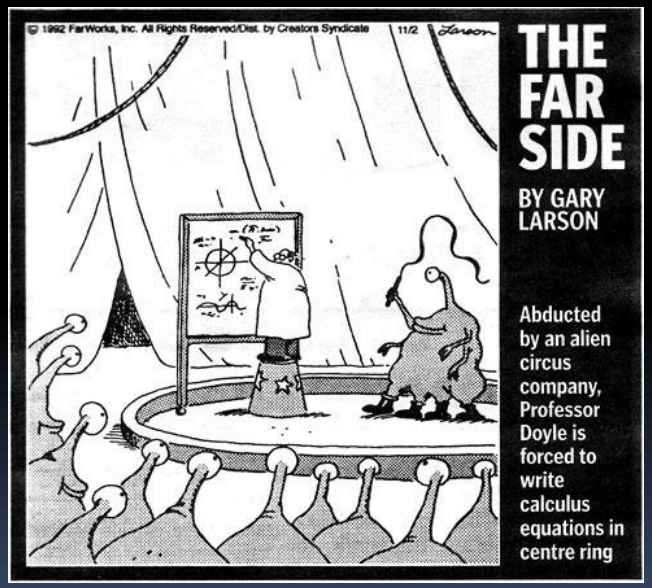
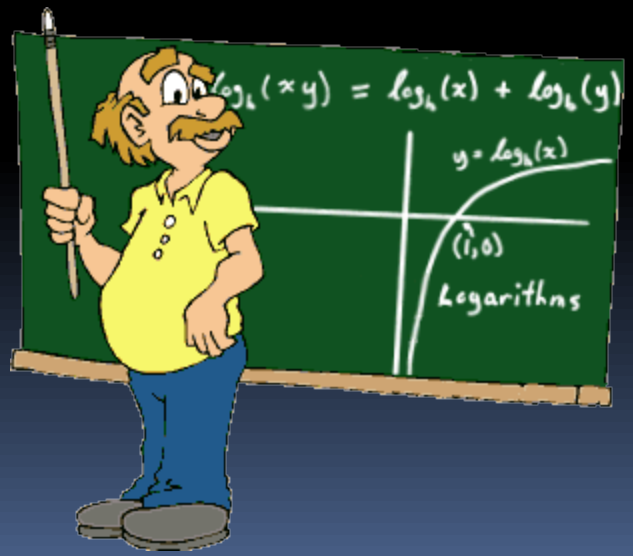


Time's Up!

MQ9. Logarithmic Progressions

- The numbers $log_{14}48,\ log_{14}a,\ log_{14}b,\ log_{14}c,\ log_{14}1875$ are in arithmetic progression. What is a? The correct answer MUST be an integer.



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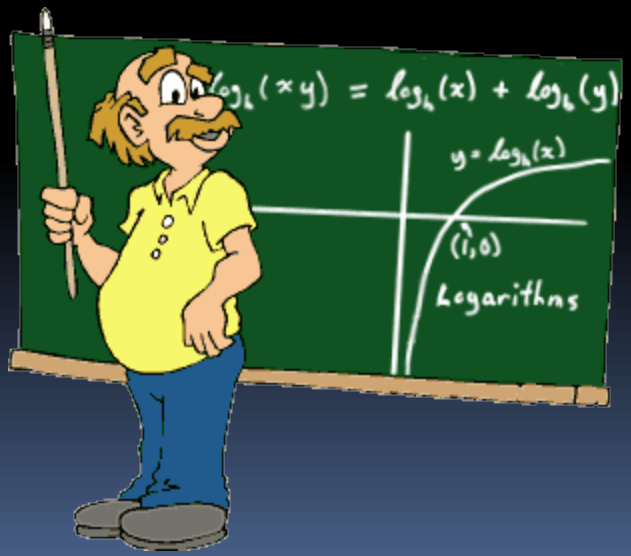

Let r be the ratio of the progression, then $\log_{14} 1875 = \log_{14} 48 + 4r$ so

$$r = \frac{1}{4}\log_{14} 1875/48 = \log_{14}(625/16)$$
$$= \log_{14}\sqrt{625/16} = \log_{14}(5/2).$$

Thus $\log_{14} a = \log_{14} 48 + \log_{14}(5/2) = \log_{14}(48 \times (5/2)) = \log_{14} 120$. Thus

$$\boxed{a = 120}$$

HQ10. The Swiss Connection

Born in Basel, Switzerland, in 1707, he became one of the greatest mathematicians of all time. A formula forever associated with him is

$$e^{i\pi} + 1 = 0$$

He was

A. Carl Friedrich Gauss
B. Pierre Laplace
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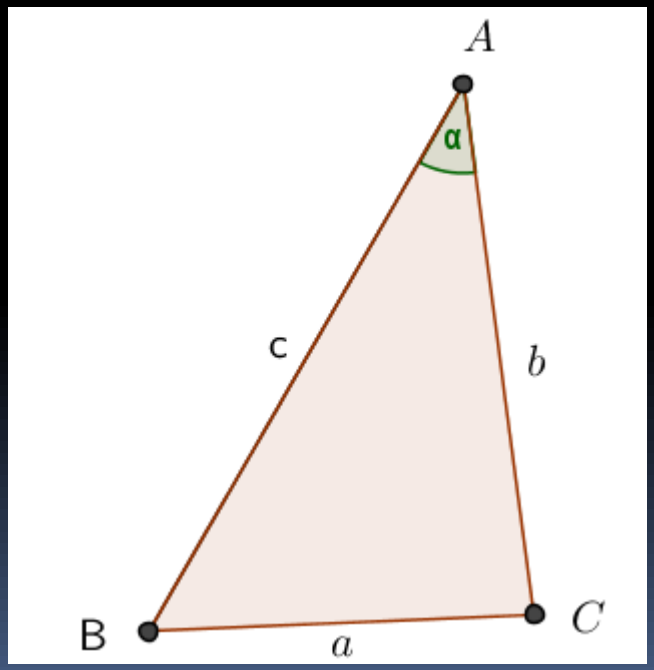
MQ10. Triangles To Remember

- In triangle *ABC*, cos α = 4/5 and the area of the triangle is 6. The smallest possible value for a is of the form $\sqrt{m}$.
- What is *m?*



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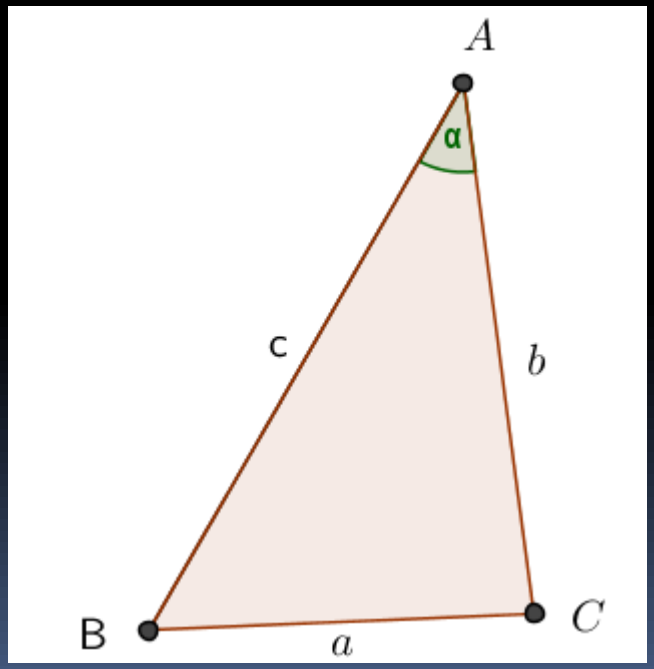

$$a^2 = b^2 + c^2 - 2bc\cos\alpha = b^2 + c^2 - \frac{8}{5}bc.$$

The area A is given by

$$A = \frac{1}{2}bc\sin\alpha = \frac{1}{2}bc\sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{3}{10}bc, \quad \text{so} \quad bc = 20.$$

So $a^2 = b^2 + c^2 - 32$; now comes a little trick; recall $bc = 20$,

$$a^2 = b^2 + c^2 - 2bc + 2bc - 32 = (b-c)^2 + 8,$$

which is smallest when $b = c$, and then $a^2 = 8$.

$$\boxed{a^2 = 8}$$

HQ11. Mechanical Calculator

- A great mathematician, he made important contributions to the theory of fluids, and invented the first mechanical calculator. He was

A. Evangelista Torricelli.
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The pascaline

MQ11. Dividing Factorials

- If n is a non-negative integer one defines n! (n factorial) by
- $0! = 1$
- $n! = n \times (n-1) \times \cdots \times 2 \times 1$.
- (So $1! = 1$, $2! = 2$, $3! = 6$, etc.)
- How many integers n, $1 \leq n \leq 50$ have the property that n does not divide $(n-1)!$?

MQ11. Dividing Factorials

- How many integers n, $1 \le n \le 50$ have the property that n does not divide $(n-1)!$?

Suppose p is prime and p^k divides n. If $k = 1$, p does not appear in the list $1, 2, 3, \ldots, p-1$, so it cannot divide $(p-1)!$. Suppose now $k \geq 2$. In this case $p \neq p^{k-1}$, both appear in the list $1, 2 \ldots, p-1$, so p^k divides $(n-1)!$. If $k = 2$, then p is in the list. If $2p < n$, so is $2p$ and p^2 divides $(n-1)!$. Can $2p \geq n$ when p^2 divides n? Since $n \geq p^2$, this implies $2p \geq p^2$, so $2 \geq p$, The only such case is $p = 2$, $n = 4$. We see that the numbers in question are precisely the prme numbers and 4:

$2, 3, 4, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.$

A total of $\boxed{16}$ numbers.