MATHEMATICAL DIVERSIONS

Math Trivia Challenge

FAU Math Day 2009

Warm Up Question

- Trigonometry is so called because

A. It was invented by Stamos Trigonometrios in the year 401 A.D.
B. It deals (at least originally) with the study of angles of triangles
C. It is part of the metric movement; it used to be called "Try going metric."
D. They couldn't think of a better name.
E. None of the above.

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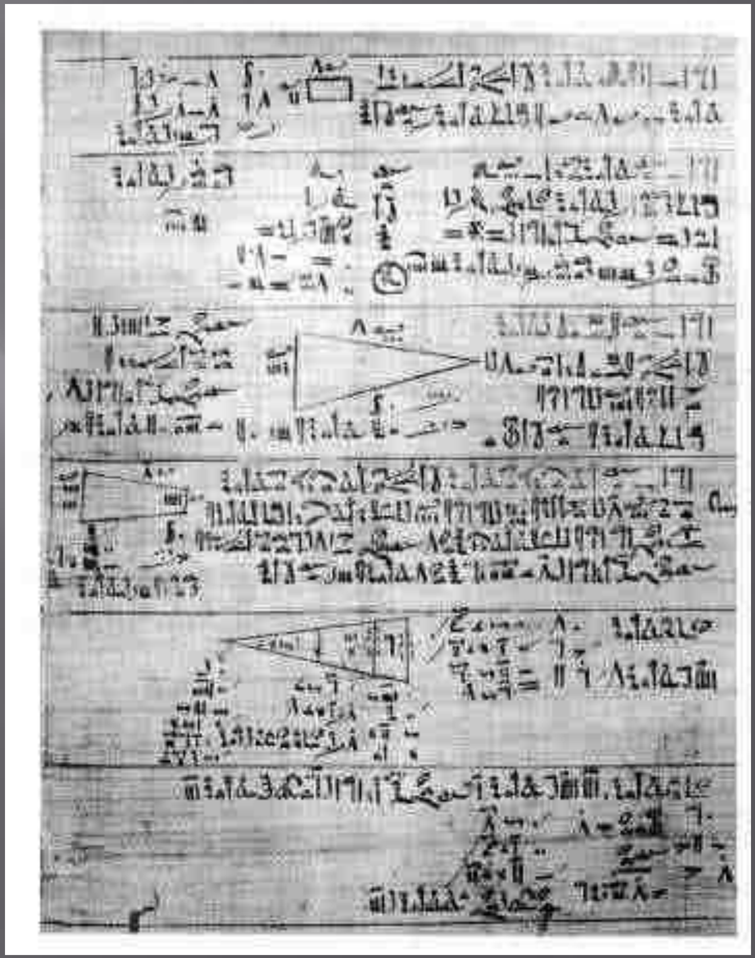
Explanation

- The word comes from the Greek; *tri* is of course three, *gonos* is angle, and *metry* has to do with measurement.

The Challenge Begins!

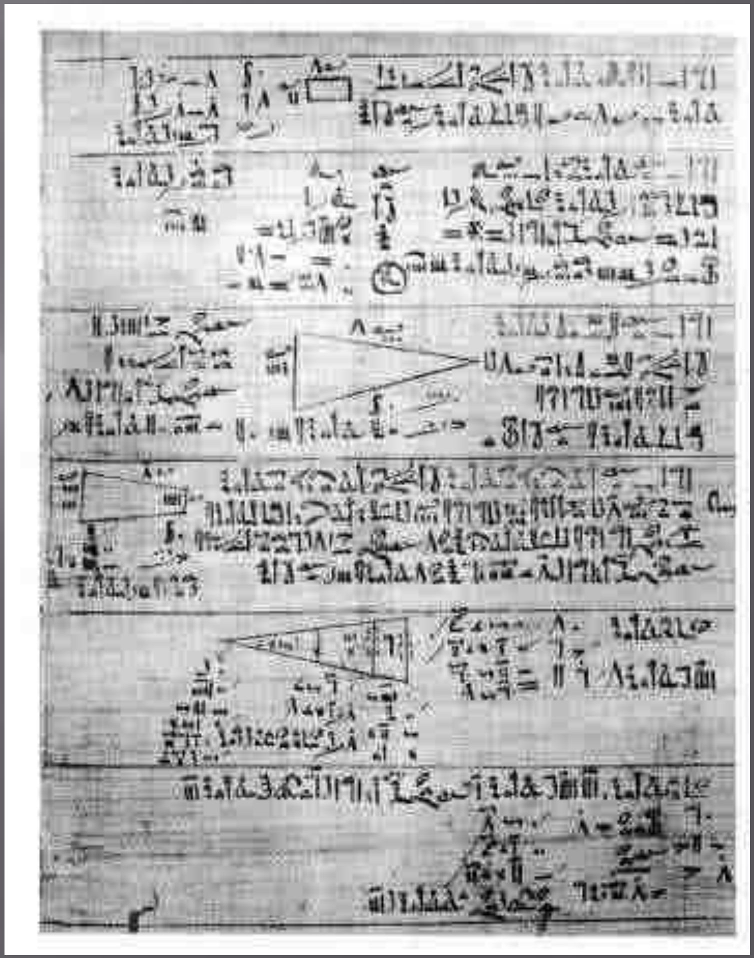
In the days of the Pharaohs

- One of the oldest mathematical documents still in existence is the Rhind Papyrus. It dates from 1650 BC, 3659 years ago; it is so named because the papyrus (an ancient form of paper) was bought by an Egyptologist of name Alexander Henry Rhind in 1858.



In the days of the Pharaohs

- Rhind donated it to the British Museum, where it remains. It is also known as the Ahmes papyrus because the writer calls himself by that name. The scribe Ahmes states that he is just copying from a much earlier work. The document contains 85 problems and their solutions.



Question 1

- This is Problem 79 from the Rhind Papyrus, going to St. Ives like an Egyptian.

- There are seven houses; in each house there are seven cats; each cat kills seven mice; each mouse ate seven grains of barley; each grain of barley would have produced seven *hekats*. What is the sum of all the enumerated things?

Houses:	7
Cats:	$7^2 = 49$
Mice:	$49 \times 7 = 343$
Grains:	$343 \times 7 = 2401$
Hekats:	$2401 \times 7 = 16807$
Total:	19607

The Answer is 19607.

From India, where 0 was invented

- Sometime between the years 400 AD and 900 AD, Indian mathematicians invented the number 0, our numerical system of base 10, and began working with negative numbers. The greatest mathematician of that time is Brahmagupta (598-670) who is still remembered in "Brahmagupta's formula" for the area of a cyclical quadrilateral. Our problem is from a later mathematician, Mahavira (active around 850).

Question 2

One fourth of a herd of camels was seen in the forest; twice the square root of that herd had gone to the mountain slopes; and 3 times 5 camels remained on the riverbank. What is the numerical measure of that herd of camels?

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ANSWER: 36 camels

Let x be the number of the herd and let $y = \sqrt{x}$.

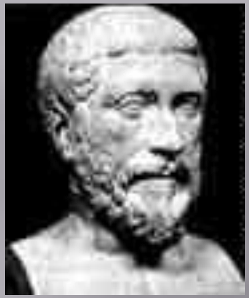
- Camels in forest: $\frac{1}{4}x = \frac{1}{4}y^2$.
- Camels on mountain slopes: $2\sqrt{x} = 2y$.
- Camels on riverbank: $3 \times 5 = 15$.

Adding up,

$$y^2 = x = \frac{1}{4}y^2 + 2y + 15; \quad \text{that is,} \quad \frac{3}{4}y^2 - 2y - 15 = 0.$$

The quadratic equation has two solutions, 6 and $-10/3$. The second solution being impossible, we get $y = 6$, $x = 6^2 = 36$.

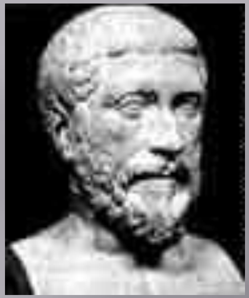
Pythagoras and his friends



The Pythagoreans were a secret society flourishing ca. 600-400 BC They called themselves followers of Pythagoras of Samos, who may or may not have existed. The most important achievement of the Pythagoreans was the discovery of irrational numbers, specifically: The hypotenuse of a right triangle of legs of length 1 is incommensurable with the legs. Or, as we say now, the square root of 2 is irrational, cannot be expressed as a ratio of two integers.

The Pythagoreans loved numbers, they adored numbers, they said "Everything is number." A property that amazed them was the existence of what they called *amicable* or *friendly* numbers. A pair of numbers m, n is an amicable pair if each is the sum of the proper divisors of the other one.

Pythagoras and his friends



Their only example was the pair 220, 284.

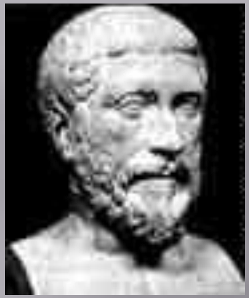
The proper divisors of 220 are: 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110.

$1+2+4+5+10+11+20+22+44+55+110 = 284.$

The proper divisors of 284 are: 1, 2, 4, 71, 142.

$1+2+4+71+142 = 220.$

Question 3



It is known that 1184 is a member of an amicable pair.

It is feeling lonely. Find its friend.

The answer is 1210

- To find the divisors of 1184 we can start seeing that it is even, see how many powers of 2 divide it. 1184/2 = 592, 592/2 = 296, 296/2 = 148, 148/2 = 74, 74/2 = 37, and 37 is prime. From this it is easy to see that the proper divisors of 1184 are:

$$1, 2, 4, 8, 16, 32, 1\times 37 = 37, 2\times 37 = 74,$$

$$4\times 37 = 148, 8\times 37 = 296, 16\times 37 = 592$$

1+2+4+8+16+32+37+74+148+296+592 = 1210

Verification

- The divisors of 1210 are:
- 1, 2, 5, 10, 11, 22, 55, 110, 121, 242, 605.
- Adding:
- 1+2+5+10+11+22+55+110+121+242+605 = 1184
- (1184,1210) is an amicable pair. It has an interesting history. Check it out!

The road to perfection

- The Pythagoreans also considered perfect numbers. A number is *perfect* if it is its own friend. Like 6. 6 needs no friend; the proper divisors of 6 are 1, 2, 3 and 1+2+3 = 6. The next perfect number is 28.
- The proper divisors of 28 are 1, 2, 4, 7, 14, and
- 1+2+4+7+14 = 28.

The road to perfection

- Euclid proved that all numbers of the form
  $2^{n-1}(2^n - 1)$
are perfect, as long as $2^n - 1$ is prime .
Notice $6 = 2^1(2^2 -1)$ and $2^2 -1 = 3$ is prime.

$28 = 2^2(2^3 -1)$ and $2^3 -1 = 7$ is prime.

Euler proved that all even perfect numbers are of this form, power of 2 times the following power of 2 minus 1, as long as the power of 2 minus 1 was prime.

Question 4

- Which of the following numbers is perfect:

A. 360
B. 594
C. 496
D. 672
E. Nobody is perfect.

Write A, B, C, D or E on your answer slip.

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Verification

- The quickest way of doing this is using the Euler-Euclid result. The perfect numbers 6 and 28 come from the primes $2^2 -1 = 3$, $2^3 -1 = 7$. Now $2^4 -1=15$ is not prime. But $2^5 – 1= 31$ is prime.
- $(2^4-1)(31) = (16)(31) = 496$
- The proper divisors of 496 are 1, 2, 4, 8, 16, 31, 62, 124, 248; they add up to 496.

Can perfection be odd?

Here are some facts about perfect numbers:

- Primes of the form $2^n - 1$ are known as Mersenne primes. Nobody knows if there is an infinite quantity of them. For each one of them one has an even perfect number.
- No odd perfect number has ever been found. Nobody knows if such a thing even exists.

Question 5 (Time flies)

- A clock is showing 2PM. For the next forty-eight hours you get $5 every time the two hands of the clock form a 90 degrees angle. How much money do you have after these forty-eight hours are over?

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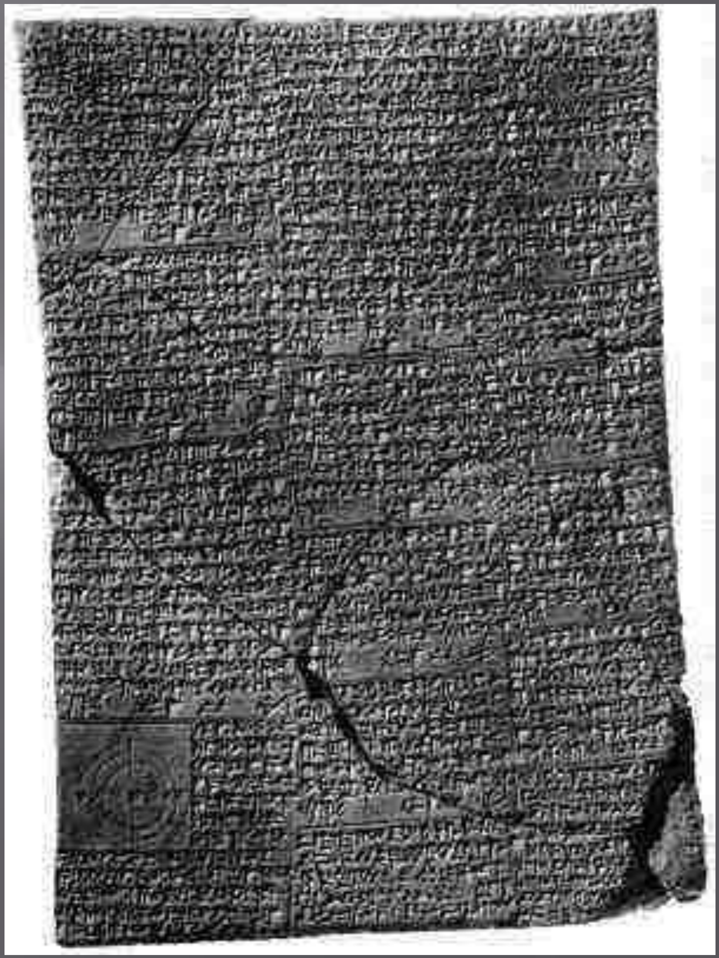
- ANSWER = $440

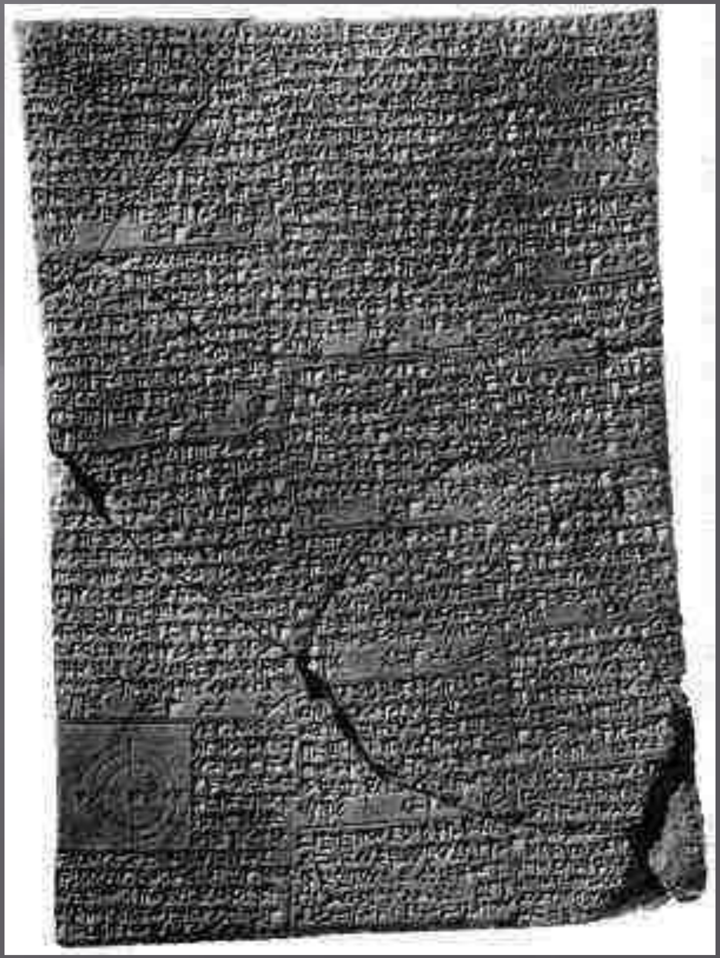
The Babylonians

How many miles to Babylon?
Three-score miles and ten.
Can I get there by candle-light?
Yes, there and back again.
If your heels are nimble and light,
You will get there by candle-light.



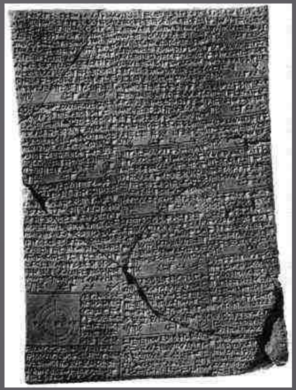
The Babylonians

The name Babylonians is given to the people living in the ancient Mesopotamia, the region between the rivers Tigris an Euphrates, modern day Iraq. They wrote on clay tablets like the one in the picture. The civilization spans several thousands of years.



Question 6 (The Babylonians)

A Babylonian tablet lists n^3+n^2 for n = 1 to 30.
Here are the first 10 entries of such a table.



n	n^3+n^2
1	2
2	12
3	36
4	80
5	150
6	252
7	392
8	576
9	810
10	1100

Tables as these seem to have been used to solve cubic equations. Solve (using the table or otherwise) for an integer solution.

$$x^3+2x^2-3136=0.$$

Hint: Set $x = 2n$.

I suppose everybody got the solution $x = 14$. Here's how the babylonians would do it, using their table. The equation was $x^3 + 2x^2 = 3136$. To get it in the form $n^3 + n^2 =$???, we set $x = 2y$ and get

$$8y^3 + 8y^2 = 3136;$$

dividing by 8,

$$y^3 + y^2 = 392;$$

from the table, $y = 7$, $x = 14$.

And now for some trivia
Or not so trivia

Question 7

- The statement that every even integer greater than 4 can be expressed as the sum of two odd prime numbers (for example, 6 = 3+3, 8 =3+5, 10 = 5+5 or 3+7, 12 = 5+7, etc.)

A. Was proved by Christian Goldbach in 1742.
B. Was proved by Leonhard Euler in 1737.
C. Has never been proved.
D. Is false for n = 2,467,892,346,778
E. Was discovered by Columbus in 1492.

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Background

- In a letter written in 1742 to Leonhard Euler, the amateur mathematician Christian Goldbach mentioned that he had observed that every even integer from 6 onward is the sum of two odd primes, and wondered if Euler had a proof. No proof has yet been found. The question is known as the *Goldbach conjecture*.

Question 8 (Pierre Fermat)

Fermat's last theorem states:

A. The equation $x^n + y^n = z^n$ has no solutions if n is a positive integer > 2.
B. The equation $x^n + y^n = z^n$ has no solutions in positive integers if n is a positive integer > 2.
C. If positive integers x, y, z solve the equation $x^n + y^n = z^n$, then n must be prime.
D. If positive integers x, y, z solve the equation $x^n + y^n = z^n$, then n must be even.
E. I shall never prove another theorem.

Pierre Fermat
1601?-1665

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Shuffling along

Suppose given a deck of 52 cards face down on a table. It doesn't matter how the cards look; all that matters is that no two should be alike. In fact, it is best to think that they are numbered from 1 to 52. Originally suppose the bottom card is the card numbered 1, the top card is numbered 52. Suppose you shuffle the deck as follows: You split the deck into two equal parts and then perfectly interlace the cards, so that now the cards appear in the order (from bottom to top) 27, 1, 28, 2, 29, 3,. . . , 25, 52, 26.

Suppose you keep shuffling that way. After how many shuffles is the deck back to its original order?

Before answering this question, we should try an easier one.

Question 9-Shuffling along

- Suppose first that we have a deck of only 8 cards, numbered from 1 to 8, originally in the order
- 1, 2, 3, 4, 5, 6, 7, 8.
- After the first shuffle, they are in the order
- 5, 1, 6, 2, 7, 3, 8, 1.
- One more shuffle, and we get
- 7, 5, 3, 1, 8, 6, 1, 2.
- In how many shuffles will the deck be again in its original order?

6 shuffles.

- One thing one can observe is that the card that was in position k, after the shuffle ends in position 2k mod 9. This suggests or shows that the number of shuffles that take the deck back to its original position is the first n such that $2^n = 1 \mod 9$. That number turns out to be 6. One might remember here that by Euler's Theorem, $2^{\phi(n)} \equiv 1 \pmod{n}$
- if n is odd; $\phi(n)$ is Euler's phi-function; $\phi(9) = 6$
- $\phi(p) = p - 1$ if p is prime

Question 10-Shuffling along

We return to our deck of 52 cards. We shuffle as explained. How many shuffles does it take to get the original order back?

A. 6
B. 49
C. 52
D. 53
E. Fifty six thousand gazillions.

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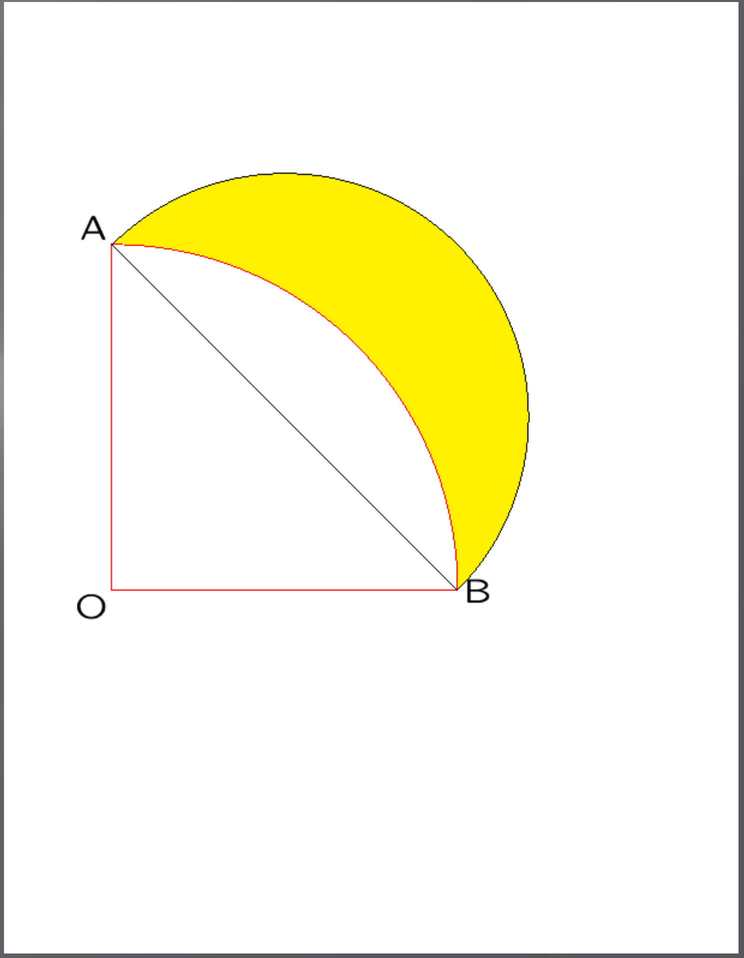
Because 53 is prime, we have $\phi(53) = 52$, so the number of shuffles must be a divisor of 52. Of the choices given, only 52 fits the bill.

Back to antiquity, Question 11

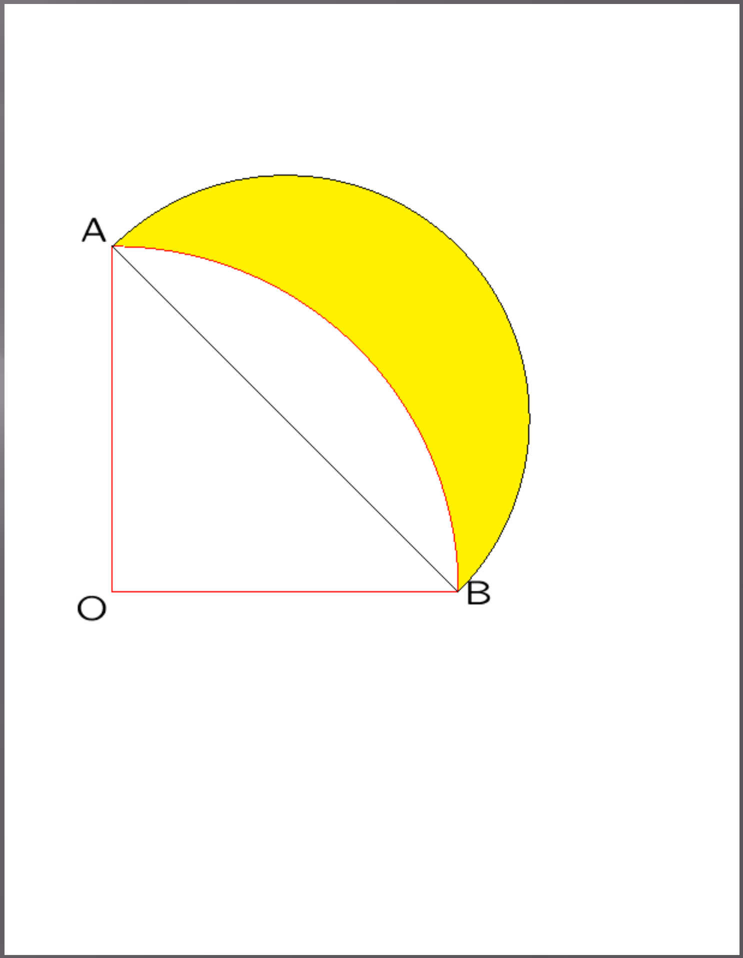
Hippocrates of Chios (ca. 440 BC, not to be confused with the more famous Hippocrates of the Hippocratic oath) found the area of some figures bounded by circles; they are known as the l*unes* of Hippocrates (from the French word for moon). One is shown here.



Question 11

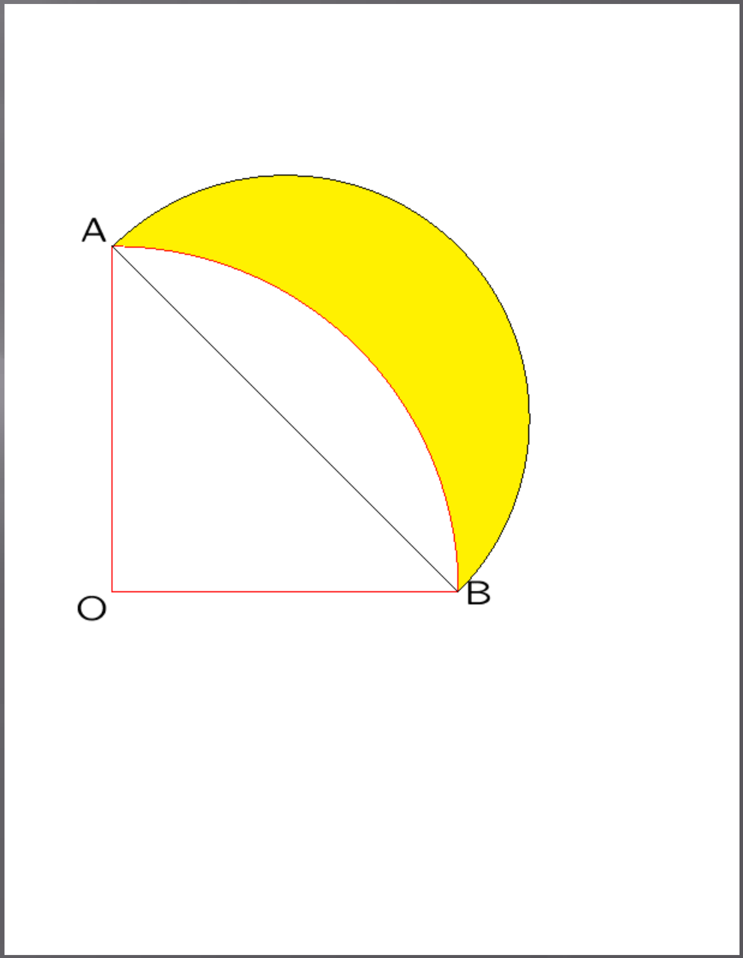
On the quarter circle AOB, use the secant AB as the diameter of a semicircle as shown in the figure. The lune is the figure in yellow, bounded by the semicircle and the quarter circle.

If $|OA| = 4$, compute the area of the lune.



Question 11

The answer is 8; the same area as the triangle AOB. The amazing thing is that no π is involved.



Question 12

- The word *algorithm*

A. Was invented by Euclid.
B. Comes from an old Indian Vedic text.
C. Is based on the name of a Buddhist mathematician who used to dance to mathematics.
D. Is based on the name of a mathematician from the Muslim world.
E. Is of Greek origin, but not invented by Euclid.

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Explanation

- Abu Jafar Muhammad ibn Musa Al-Khwarizmi (ca. 790-840) may have been born in central Asia, or near Baghdad, but he was active in Baghdad during the most brilliant period of the caliphate. He wrote a very influential book whose title contained the words "al-jabr", meaning completion.

Explanation

- It deals with the solution of equations, and people began to do "al-jabr" in the style of Al-Khwarizmi, or algebra algorithmically.

Question 13

- In the year 2000, the Clay Institute, a mathematics research institute based in Massachusetts, set up a committee of eminent mathematicians to identify the most important open problems in mathematics, those that would drive research for the next 1,000 years, the *Millennium Problems.* The solver of any one of these problems will be rewarded by a million dollar prize.

Question 13

- The Millenium Problems identified by the committee were:

1. The Birch and Swinnerton-Dyer Conjecture.
2. The Hodge Conjecture.
3. Navier Stokes equations, existence and smoothness of solutions in 3 dimensions.
4. P vs. NP.
5. The Poincare conjecture.
6. The Riemann hypothesis.
7. Yang-Mills Theory, existence and the ``mass gap.''

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Perelman and Poincare

The picture at the top is Grigori Perelman. At the bottom is Poincare. Grigori Perelman refused to accept the million dollar prize. He also refused to travel to Madrid to attend the International Congress of Mathematicians of 2006 that was to award him one of the highest honors a mathematician can receive, the *Fields medal.*
In addition, he declined accepting the Fields medal.

Question 14-Bridges

- It is known that the great mathematician Leonhard Euler solved the Konigsberg bridges problem. The problem had something to do with

A. A Russian city.
B. A Swiss industrialist.
C. A special curve minimizing certain distances.
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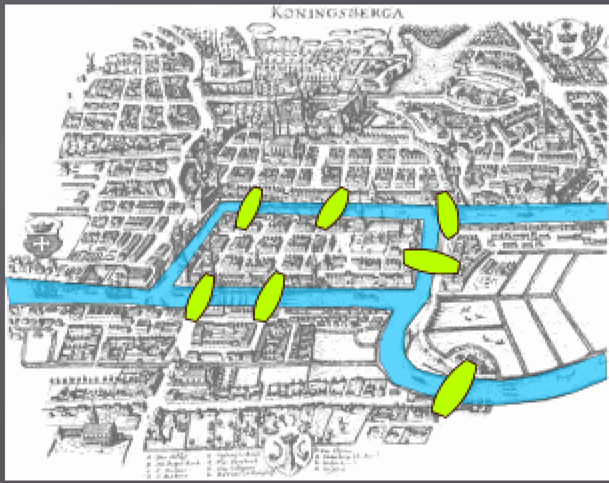
Question 15--Bridges

The city is Kaliningrad, in Russia, but it used to be called Konigsberg and was in Germany. The city lies on the river Pregel, there are two islands in the river and seven bridges connecting both sides of the mainland and the islands. The mayor of Konigsberg wanted to know if he could take a tour which would have him cross all bridges exactly once. Euler gave a very ingenious negative solution to the problem on a 1736 visit to the city. That solution is considered the birth of *graph theory*.



Question 15

- Who said:
- Students are taught advanced concepts of Boolean algebra, and formerly unsolvable equations are dealt with by threats of reprisals.

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