

MATH CIRCLE AT FAU

11/18/2017
Middle School Version

Session # 5

RULES

- Work the problems in any order. Some problems are harder than others; do what you can. **If later on you take the time to figure out how to solve the problems you could not solve, you will learn more from what you could not do, than from what you could do.**
- If you think you have finished a problem correctly, tell one of the organizers. If it is really correct, he or she will certify that it is correct.
- Don't feel shy about asking for hints.
- Don't feel shy about getting up, walking around, or talking with anybody you want to talk to.
- If you want to write on one of the whiteboards, we have markers available.

1. This is a classic question. A census-taker knocks on a door and asks the woman inside how many children she has and how old they are.

“I have three daughters, their ages are whole numbers, and the product of their ages is 36.”

“That’s not enough information,” responds the census-taker.

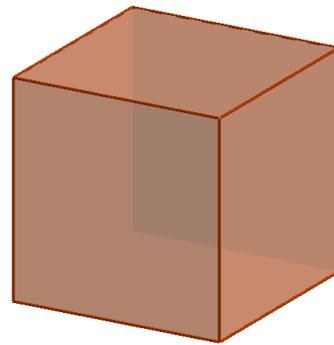
“I’d tell you the sum of their ages, but you’d still be stumped.”

“I wish you’d tell me something more.”

“O.K., my oldest daughter Anne likes dogs.”

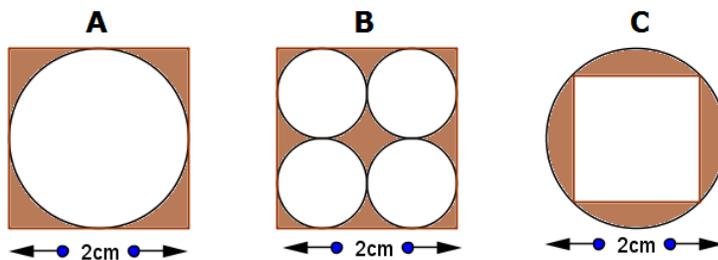
How old are the daughters?

2. A farmers market sells only baskets of apples and baskets of oranges, each for a fixed whole number of dollars. Abigail paid \$23 for five baskets of fruit. Benedict paid \$24 for four baskets of fruit. If Charles paid \$20, how many baskets of fruit did he buy?
- 3.



Jamie counted the number of edges of a cube, Jimmy counted the number of corners, and Judy counted the number of faces. They then added the three numbers. What was the resulting sum?

4. The following figures are composed of squares and circles. Which figure has a shaded region with largest area?



(A) A only (B) B only (C) C only (D) both A and B (E) all are equal

5. If 20% of a number is 12, what is 30% of that number?
6. A group of children riding on bicycles and tricycles rode past Billy Bob’s house. Billy Bob counted 7 children and 19 wheels. How many tricycles were there?
7. Ali, Bonnie, Carlo and Dianna are going to drive together to a nearby theme park. The car they are using has four seats: one driver’s seat, one front passenger seat and two back seats. Bonnie and Carlo are the only two who can drive the car. How many possible seating arrangements are there?

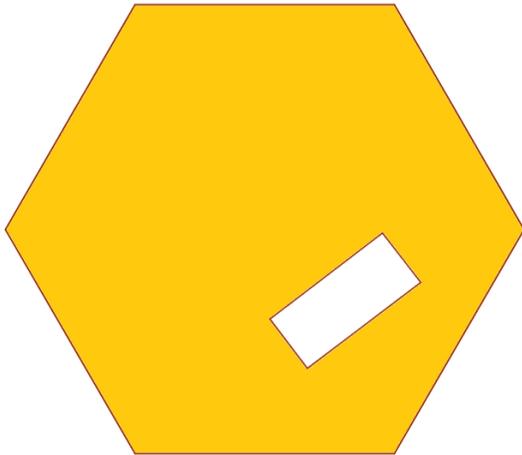
8. A *palindrome* is word, phrase or number that reads the same forward as backwards. Two famous palindromes are:

Able was I ere I saw Elba

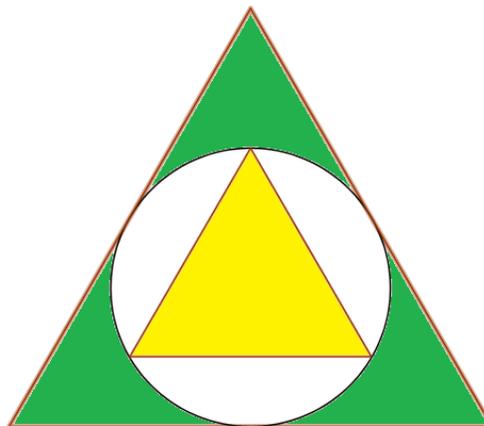
A man, a plan, a canal, Panama.

But we are doing mathematics, so we look at numbers that are palindromic, like all one digit numbers, 33, 656, 98789, etc. Here is the question: What four digit palindrome has only one prime factor? (Only one prime number divides it).

9. A rectangle has been cut out of a regular hexagon. Draw a straight line that divides the shaded figure (the hexagon minus the rectangle) into two parts of equal area. The only tool you can use here is a ruler; but you can only use it to draw straight lines.

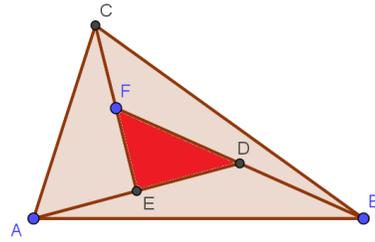


10. The picture shows an equilateral triangle inscribed in a circle inscribed in an equilateral triangle. What is the ratio between the areas of the two triangles. If T is the area of the big triangle, and S is the area of the small triangle, what is $\frac{T}{S}$? (From Martin Gardner's *Puzzle Tales*.)



11.

Segments are drawn in triangle ABC in such a way that D is the midpoint of BF , E is the midpoint of AD , and F is the midpoint of CE . If the area of triangle ABC is 1, what is the area of triangle DEF ?



12. In the figure, the area of square $WXYZ$ is 25 cm^2 . The four smaller squares have sides 1 cm long, either parallel to or coinciding with the sides of the large square. In $\triangle ABC$, $|AB| = |AC|$, and when $\triangle ABC$ is folded over side BC , point A coincides with O , the center of square $WXYZ$. What is the area of $\triangle ABC$, in square centimeters?

