MATH CIRCLE AT FAU

11/04/2017

Middle School Version

Session # 4

RULES

- Work the problems in any order. Some problems are harder than others; do what you can. **If later on you take the time to figure out how to solve the problems you could not solve, you will learn more from what you could not do, than from what you could do.**
- If you think you have finished a problem correctly, tell one of the organizers. If it is really correct, he or she will certify that it is correct.
- Don't feel shy about asking for hints.
- Don't feel shy about getting up, walking around, or talking with anybody you want to talk to.
- If you want to write on one of the whiteboards, we have markers available.

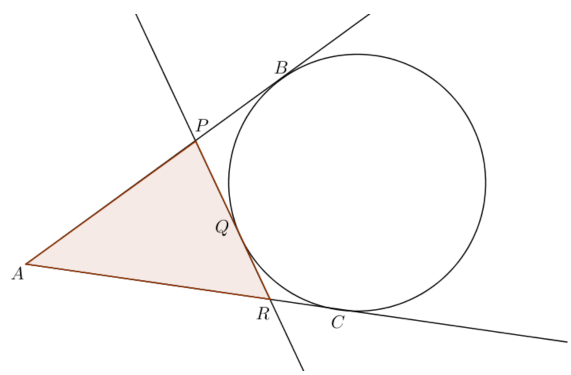
1. Every day Mr. Wooster returns from his club in the city (were he spends the day drinking tea and gossiping) by train to the train station of the town were he lives. His butler Jeeves is supposed to pick him up by car and drive him home. No time is to be wasted; Jeeves is to leave the house so he exactly arrives at the train station the moment the train pulls in, Mr. Wooster jumps into the car and they drive immediately home. One day the train arrives early, Jeeves isn't there yet, and Mr. Wooster decides to walk home. After walking for half an hour he meets Jeeves on the way to pick him up. He gets into the car and they arrive at the home 20 minutes earlier than usual. How many minutes early was the train?

2. There are 2017 representatives of four of the Middle-Earth races sitting around a round table, dwarves, elves, gnomes,and humans. Elves never sit next to dwarves, and humans cannot sit next to gnomes. Prove that at least one pair of representatives sitting next to each other must be of the same race.

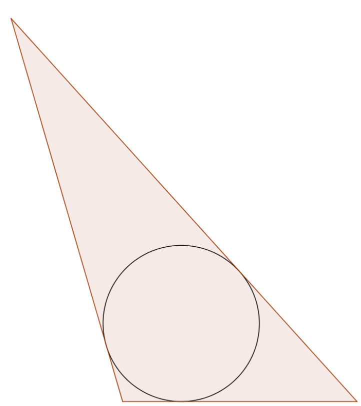
3. Suppose there are exactly 9 towns in a very small country and **all distances between the towns are different**. A person starts in each town and walks towards the **nearest** town. Prove: (a) There are two towns A and B such that a person from A walks to B, and a person from B walks to A. (b) There is a town that nobody walks to.

4. A certain country has several airfields with lots of planes in each. The distances between all of the airfields are different; no two are at the same distance. One day an airplane takes off from each field and lands on the closest airfield. Prove that at most 5 airplanes will land on each airfield.

5. From a point A outside of a circle we draw two tangents touching the circle at points B, C, respectively. We then draw a third tangent intersecting segment AB at P, segment AC at R and touching the circle at Q. If $|AB| = 20$, what is the perimeter of triangle APR? Can one even determine it from the provided data?



6. The ideas in the solution of the previous problem play a role here. The perimeter of a triangle is 75 inches, the radius of the inscribed circle is 5 inches. What is the area of the triangle?

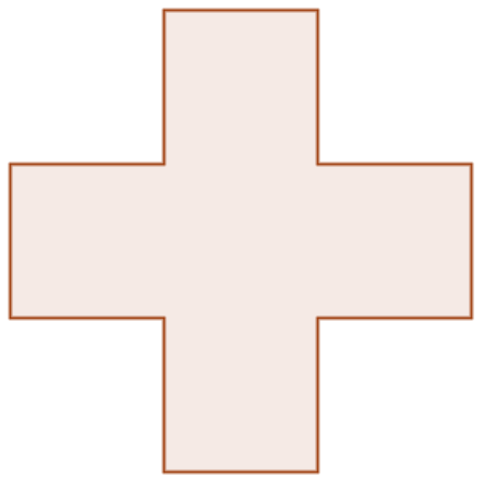


7. One of the legs of a **right** triangle has length 12 and the hypotenuse has length 13. Find the area of the inscribed circle. This should be quite easy after having done the previous problem.

8. A triangle has sides of length 9, 10, and 17. What is the radius of the inscribed circle? This can be easy if we know Heron's formula. If not, it could be hard.

9. Using only a compass and a straightedge, how can one find the *incenter* of a triangle; the center of the inscribed circle? On page 4 of this problem set you'll find a few triangles on which you can practice. The incenter must be at the same distance from each one of the sides; is there always such a point?

10. The cross pictured below, having all arms of the same length, can be divided by two cuts into four pieces that can be assembled to form a square.



On the page 5 you will find several copies of the cross to practice your cuts.

TRIANGLES AND CROSSES