

MATH CIRCLE AT FAU

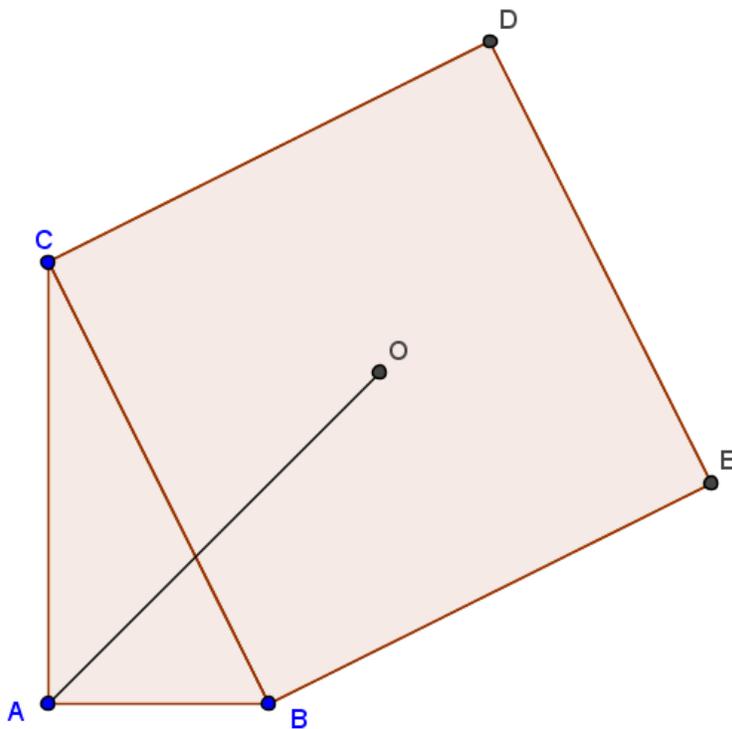
10/28/2017
Middle School Version

Session # 3

RULES

- Work the problems in any order. Some problems are harder than others; do what you can. **If later on you take the time to figure out how to solve the problems you could not solve, you will learn more from what you could not do, than from what you could do.**
- If you think you have finished a problem correctly, tell one of the organizers. If it is really correct, he or she will certify that it is correct.
- Don't feel shy about asking for hints.
- Don't feel shy about getting up, walking around, or talking with anybody you want to talk to.
- If you want to write on one of the whiteboards, we have markers available.

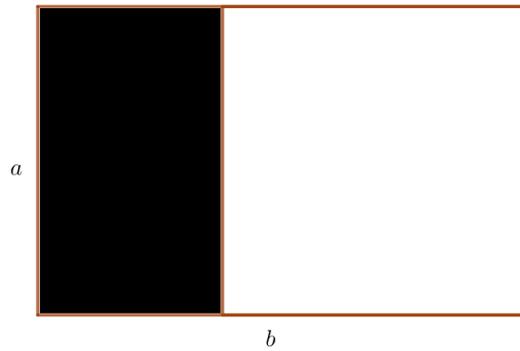
1. A boy buys trumples at 3 for 10 cents. He sells them at 5 for 20 cents. He makes a profit of 1 dollar. How many trumples did he sell? (A *trumple* is a musical instrument made by elves).
2. Helen Pythagoras placed a 25 foot ladder against a vertical wall of a building. The foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips four feet, how much will the foot of the ladder slip?
3. A grasshopper jumping along a straight line can jump 6 or 8 inches in either direction. Can it reach a point that is (a) 1.5 inches away from its original position; (b) 7 inches away; (c) 4 inches away?
4. A cardboard rectangle of area 1 is cut into two pieces along a line segment that connects the midpoints of two adjacent sides. Find the areas of the two pieces. JUSTIFY YOUR ANSWER.
5. The numbers 2^{2017} and 5^{2017} are expanded and their digits are written out consecutively on one page. How many total digits are on the page.
- 6.



The square $BCDE$ has one side coinciding with the hypotenuse of the right triangle ABC . If $|AB| = a$ and $|AC| = b$, determine the length of the segment AO from A to the center O of the square.

7. A triangle has altitudes of lengths 24, 24, and 20, respectively.
 - (a) Explain why this triangle must be isosceles.
 - (b) Find the radius of the inscribed circle.
8. 852 digits are used to number the pages of a book consecutively from page 1. How many pages does the book have?
9. A palindromic number is a whole number that reads the same both ways, for example 111, 45654 are palindromic. How many palindromic numbers are there between 10 and 10000?

10. A rectangle of dimensions $a \times b$ with $b - a < a < b$ has the property that if we snip off a square whose side is the smaller side of the rectangle, we get a rectangle that is similar to the original one:



The darkly shaded small rectangle and the large rectangle are similar. What is the ratio b/a ?