

## **MATH CIRCLE AT FAU**

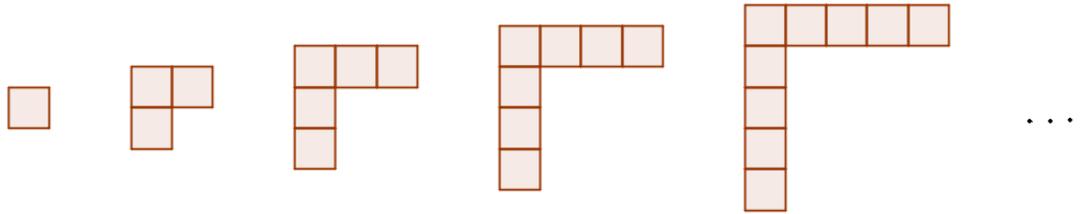
10/07/2017

Session # 2

### **RULES**

- Work the problems in any order. Some problems are harder than others; do what you can. **If later on you take the time to figure out how to solve the problems you could not solve, you will learn more from what you could not do, than from what you could do.**
- If you think you have finished a problem correctly, tell one of the organizers. If it is really correct, he or she will certify that it is correct.
- Don't feel shy about asking for hints.
- Don't feel shy about getting up, walking around, or talking with anybody you want to talk to.
- If you want to write on one of the whiteboards, we have markers available.

1. (From the Moscow Math Circle book) Here is a series of figures.



The first consists of one square. How many squares are there in the 100th figure? How many squares are there in the first 100 figures altogether?

2. What is  $1 + 3 + 5 + \dots + 199$ ?
3. An *arithmetic progression* is a (finite) sequence of numbers such that any two consecutive numbers in the sequence differ by the same amount. For example:

$$1, 2, 3, 4, 5, 6$$

is an arithmetic progression, any two consecutive numbers differ by 1.

$$0, 3, 6, 9, 12, 15, 18, 21, 24, 27$$

is an arithmetic progression; any term differs from the preceding one by 3.

Generally speaking, an arithmetic progression is a sequence of the form

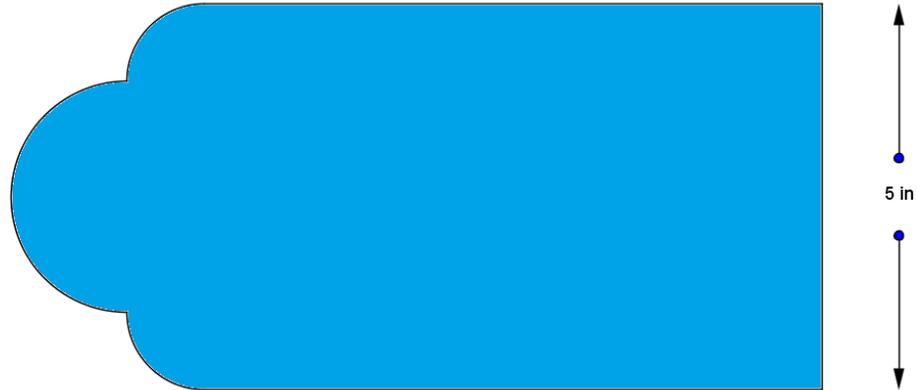
$$a, a + d, a + 2d, \dots, a + (n - 1)d.$$

We call  $d$  the *difference*,  $n$  is the number of terms. Here are some progression questions:

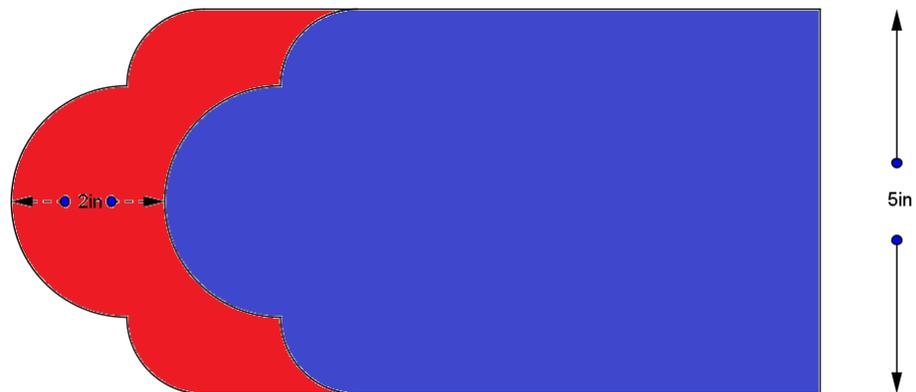
- (a) Compute  $1 + 2 + 3 + \dots + 99 + 100$ .
- (b) Compute  $2 + 7 + 12 + 17 + 22 + 27 + 32 + \dots + 202$ .
- (c) An arithmetic progression consists of 100 terms; its first term is 3 and the difference is 2. Find its sum.



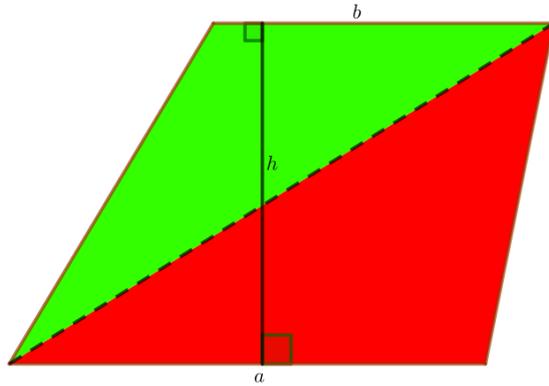
7. The front door of a mansion has an opening of the shape shown below, covered by a metal plate that can slide back when Paunchy the butler needs to look out and identify a visitor. It slides into the door. Here is how things look when the plate completely closes the opening. The shape of the left edge is made up of two quarter circles and one half circle.



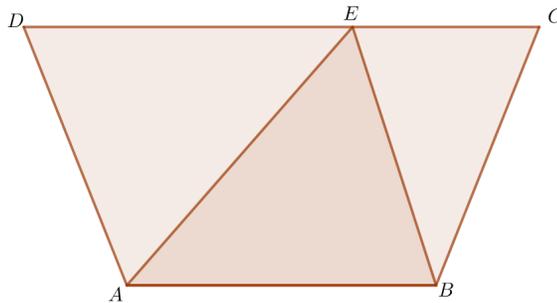
Paunchy slides the lid back by 2 inches. What is the area of the opening that has been uncovered? (Colored red in the picture)(A power point simulation will be shown.)



8. A *trapezoid* is a quadrilateral with two parallel sides, frequently called the bases of the trapezoid. I assume you know a formula for the area of a triangle (if you don't ASK!). You may or may not know a formula for the area of the trapezoid in terms of the length of the bases  $a, b$  and the altitude  $h$ . If you know it, explain why it is true. If you don't know it, figure it out from the picture below.



9. The quadrilateral  $ABCD$  is a trapezoid with  $AB$  parallel to  $CD$ . Side  $AB$  has length 12 units, side  $BC$  has length 20 units and the area of triangle  $AEB$  is 60 square units. Find the area of the trapezoid.



- 10.

A circle of center  $O$  and radius 2 cm contains three smaller circles as shown in the picture; two of them touch the outer circle and touch each other at  $O$ , and the third touches each of the other circles. Find the radius of this last circle in centimeters.

