

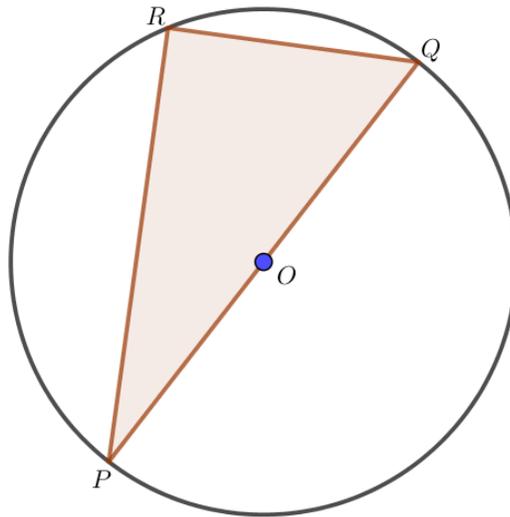
MATH CIRCLE AT FAU

09/23/2017

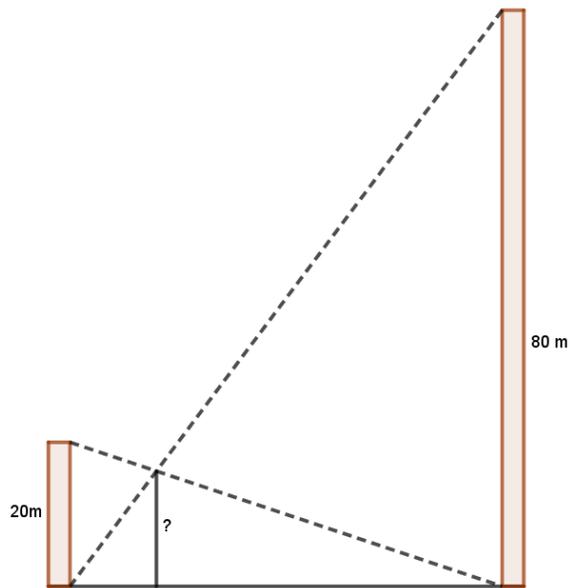
Session # 1

Problems marked with a star (\star) are from *Lectures and Problems, A Gift to Young Mathematicians*, by V.I. Arnold, ©2015 Mathematical Sciences Research Institute.

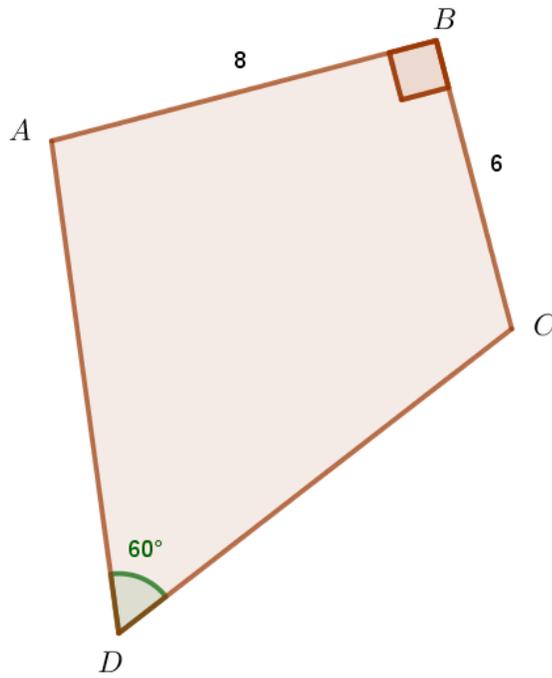
1. \star Mary and Murray want to buy their mother a cake for her birthday. Murray was seven dollars short of the price and Mary was one dollar short. They combined their money to buy the cake, but even then they did not have enough. How much did the cake cost? (The price of the cake is in dollars, no cents.)
2. \star A bottle with a cork costs \$1.10 while the bottle alone costs 10 cents more than the cork. How much does the cork cost?
3. \star Two people left at dawn, at the exact same time, one traveling from A to B , the other one from B to A . They travel at a constant speed, without stopping. They meet at noon. The first one arrives at B at 4 p.m., the second one arrives at A at 9 p.m.
At what time was dawn that day?
4. \star What is the maximum area that a right triangle with a hypotenuse that is 10 inches long can have?
5. PQ is the diameter of a circle with center O ; R is a point on the circumference. If $PO = OQ = QR = 1$, find PR .



6. Two vertical poles 20m and 80m high stand apart on a horizontal plane. Find the height in meters of the point of intersection of the lines joining the top of each pole to the foot of the other.



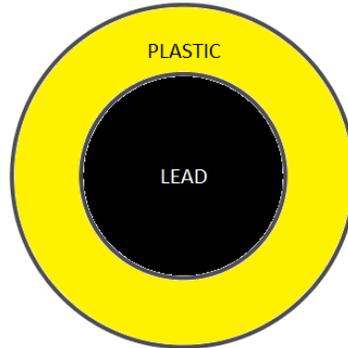
7. Something easy now. An equilateral triangle has sides of length 6. What is its area.
8. $ABCD$ is a quadrilateral, $AD = CD$, $AB = 8$, $BC = 6$, $\angle ADC = 60^\circ$ and the angle at B is a right angle. Find the area.



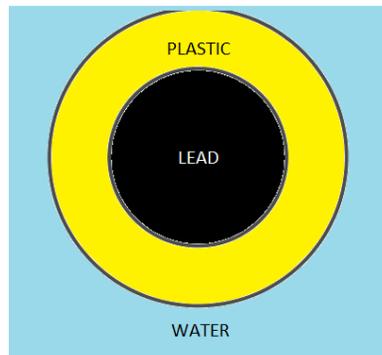
9. And now for something completely different. For this problem you need to know the the volume of a ball (sphere) of radius r is

$$V = \frac{4}{3}\pi r^3.$$

A plastic ball of radius 12 inches has been hollowed out and filled with lead:



So we have a ball of lead covered by plastic. This ball when placed in water sinks so that is even with the surface of water. If the specific gravity of lead is 10 times that of water, of the plastic is $1/36$ that of water, what is the radius of the lead ball? (The specific gravity of water can be taken as 1).



The answer should be the root (square root?, cubic root?) of a fraction times an integer.