1) Compute the cross product $u \times v$ between the following vectors:

- $u = (1, 2, 3)$ $v = (1, 0, 1)$
- $u = (1, 1, 1)$ $v = (0, 0, 0)$
- $u = (1, 2, 3)$ $v = (4, 5, 6)$
- $u = (\cos \theta, 0, 1)$ $v = (0, \sin \theta, 1)$, $\theta \in \mathbb{R}$
- $u = a \times b$, $v = b \times a$, $a, b \in \mathbb{R}^3$
- $u = e^{42}(1, 0, 0)$ $v = e^{51}(0, 1, 0)$
- $u = (f(t), g(t), 0)$ $v = (g(t), -f(t), 0)$, $f, g : \mathbb{R} \to \mathbb{R}$

2) Compute the cosines of the angles between the following vectors:

- $u = (1, 2, 3)$ $v = (1, 0, 1)$
- $u = (f(t), g(t), 0)$ $v = (g(t), -f(t), 0)$, $f, g : \mathbb{R} \to \mathbb{R}$
- $u = (\cos \theta, \sin \theta)$ $v = (0, 1)$, $\theta \in \mathbb{R}$
- $u = (\sin \theta, \cos \theta)$ $v = (0, 1)$, $\theta \in \mathbb{R}$
- $u = (\cos \theta \sin \psi, \cos \theta \cos \psi, \sin \theta)$, $v = (1, 0, 0)$, $\theta, \psi \in \mathbb{R}$
- $u = (0, 0, 0)$ $v = (0, 0, 0)$
- $u = (1, 0, 0)$ $v = (0, 0, 0)$

3) Find $\text{proj}_u(v)$ (the projection of $v$ over $u$) for the following vectors:

- $u = (1, 2, 1)$ $v = (0, 0, 1)$
- $u = (1, 2, 1)$ $v = (0, 0, 0)$
- $u = (5, 1, 10^{99999})$ $v = (1, 0, 0)$
- $u = (5, 1, 10^{99999})$ $v = (0, 1, 0)$
- $u = (5, 1, 10^{99999})$ $v = (0, 0, 1)$
- $u = (5, 1, 10^{99999})$ $v = (5, 1, 10^{99999})$
- $u = (f(t), g(t), 0)$ $v = (g(t), -f(t), 0)$, $f, g : \mathbb{R} \to \mathbb{R}$
4) Find parametric equations for the line in \( \mathbb{R}^2 \) whose equation is \( x = 2y + 1 \).

5) Find parametric equation for the plane in \( \mathbb{R}^3 \) whose equation is \( z - y + 2x - 1 = 0 \).

6) Find parametric equation for the line in \( \mathbb{R}^3 \) whose equations are \( x = 2y + 1 \) and \( z - y = 3 \).

7) Find parametric equation for the line in \( \mathbb{R}^2 \) whose equation is \( y = 3x - 4 \).

8) Find parametric equation for the plane in \( \mathbb{R}^3 \) whose equation is \( y = 3x - 4 \).

9) Find parametric equation for the line obtained by intersecting the planes \( z = x - y \) and \( x - 1 + 2y + z = 0 \).

10) Find parametric equation for the lines obtained intersecting the cylinder \( x^2 + y^2 = 1 \) with the plane \( x - 2y = 0 \).

11) Find parametric equation for the plane passing through \( P = (1, 2, 3) \) and perpendicular to \( (1, 0, 1) \).

12) Find parametric equation for the line passing through \( P = (1, 2, 3) \) and perpendicular to both \( (1, 0, 1) \) and \( (0, 1, 2) \).

13) Find an equation for the plane whose parametric equations are:

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}
\]

14) Find an equation for the line in \( \mathbb{R}^3 \) whose parametric equations are:

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}
\]

15) Find parametric equations for the tangent line \( \ell \) to the curve:

\( \gamma(t) = (1, t, t^4) \)

at the point \( \gamma(2) \). Find also the equations for two planes whose intersection is the line \( \ell \).

16) Find parametric equations for the tangent line to the helix with parametric equations:

\( x = 2\cos t, \quad y = 2\sin t, \quad z = t^2 \)

at the point \( (0, 1, \frac{\pi^2}{4}) \).

17) Compute the length of the curve:

\( \gamma(t) = 2\cos t, \quad y = 2\sin t, \quad z = t \)

defined on the interval \([0, 3] \).
18) Compute the length of the plane curve:

\[ \gamma(t) = (e^{-t} \cos t, e^{-t} \sin t) \]

defined on the interval \([0, \infty)\).

19) Find parametric equations for the tangent line to the curve:

\[ x = t, \quad y = e^{-t}, \quad z = 2t - t^2 \]

at the point \((0, 1, 0)\).

20) Find the length of the curve:

\[ \gamma(t) = \left(2t, t^2, \frac{t^3}{3}\right), \quad 0 \leq t \leq 3 \]

21) Reparametrize the curve with respect to the arc-length measured from the point where \(t = 0\) in the direction of increasing \(t\):

\[ \gamma(t) = (e^{3t} \cos(3t), 3, e^{3t} \sin(3t)) \]

22) Find the curvature of the following curves:

\[ \gamma(t) = (e^t, 0, \cos t) \]

\[ \gamma(t) = (e^t, 1, \cos t) \]

\[ \gamma(t) = (t, 0, 1) \]

\[ \gamma(t) = (1, 0, t^3) \]

\[ \gamma(t) = (t, t^2, t^3) \]

23) Find the equation of a parabola whose curvature at the origin is 3.

24) Find the equations of the normal and osculating plane of the curve of intersection of the two surfaces \(x = 2y^2\) and \(z = x^2\) at the point \((2, 1, 4)\).

25) Compute the indicated partial derivatives for the functions:

\[ f(x, y) = \cos(x + y), \quad f_{xy} \]

\[ f(x, y) = \cos(x^2 + y), \quad f_{yx} \]

\[ f(x, y) = e^{\cos(x+y)}, \quad f_{yx} \]

\[ f(x, y) = x^2 + 2xy - \cos(g(x)), \quad f_{xx} \]
26) Find the limit, it it exists, or show that the limit does not exist:

\[
\lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^2 + y^2} \\
\lim_{(x,y) \to (2,1)} \frac{4 - xy}{x^2 + 3y^2} \\
\lim_{(x,y) \to (0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2} \\
\lim_{(x,y) \to (0,0)} \frac{xy(e^{x+y} - 1)}{x^2 y + 2xy}
\]

27) Find the following limit (hint: use Taylor expansion...):

\[
\lim_{x \to 0} e^{(\sin x)^n}
\]

28) Find the equation for the tangent plane to the given surface \( S \subset \mathbb{R}^3 \) at the specified point \( P \):

\[
S = \{ z = x^2 + y^2 - \cos x^2 \}, \quad P = (0,0,0) \\
S = \{ x^2 + y^2 - 1 = 0 \}, \quad P = (0,1,10^6) \\
S = \{ \text{graph of the function } f(x,y) = e^{xy} - x^2 - 1 \}, \quad P = (0,0,0) \\
S = \{ \text{graph of the function } f(x,y) = (2x + 3)(x^2 - y^2) \}, \quad P = (0,0,0) \\
S = \{ \text{graph of the function } f(x,y) = (2x + 3)(x - y) \}, \quad P = (0,0,0)
\]

29) Find the linear and quadratic approximation\(^1\) of the following functions at the given point \( Q = (x,y) \):

\[
f(x,y) = x^2y^2 + xy + 2x + y + 4x^2 - 3, \quad Q = (0,0) \\
f(x,y) = e^{\cos x \sin y} + \frac{e^{(\sin xy)^n}}{1 + (xy)^n}, \quad Q = (0,0) \\
f(x,y) = x^n + y^m, \quad Q = (0,0) \quad \text{and} \quad n, m > 2. \\
f(x,y) = x^n + y^m + x^2 + 5y + \cos(2y)^3, \quad Q = (0,0) \quad \text{and} \quad n, m > 2.
\]

30) Find the gradient of the given function \( f \), evaluate it at the point \( P \) and find the rate of change of \( f \) in the direction of the given vector \( u \):

\[
f(x,y) = \cos(2x + 3y^2), \quad P = (0,0), \quad u = (3,2) \\
f(x,y) = e^{\sin(xy)x^2}, \quad P = (0,1), \quad u = (1,1) \\
f(x,y) = x^4 + y^5 + 57x - y, \quad P = (0,0), \quad u = (1,5) \\
f(x,y,z) = x^n y^m z^n + \cos x, \quad P = (0,0), \quad u = (1,1), \quad n > 2 \\
f(x,y,z) = x^n y^m z^n + y \cos x, \quad P = (0,0), \quad u = (1,1), \quad n > 2
\]

\(^1\)See pag. 956-957 of the book for a review for the case of two variables.
\[ f(x, y, z) = x^n y^n z^n + y \cos(zx) + 2 \sin z, \quad P = (0, 0), \quad u = (1, 1), \quad n > 2 \]

31) Suppose \( g : \mathbb{R}^2 \to \mathbb{R} \) is a function with \( \nabla g(0, 0) = (1, 3) \). Compute the gradient of:

\[ f(x, y) = \cos(x + 2y) + g(x, y) \]

at the point \((x, y) = (0, 0)\).

32) Suppose \( g : \mathbb{R}^2 \to \mathbb{R} \) is a function whose linear approximation is:

\[ g(x, y) = 2 + 3x - 4y + \epsilon(x, y) \]

where \( \epsilon(x, y) \) is a quadratic error. Compute the gradient of:

\[ f(x, y) = e^{g(x, y)} + \cos(g(x, y) - 2) \]

at the point \((x, y) = (0, 0)\).

33) Find the local minimum, maximum and saddle points of the functions:

\[
\begin{align*}
  f(x, y) &= 2x^2 + xy + y^2 + 3y \\
  f(x, y) &= (x + y)e^{-2x^2 - sy^2} \\
  f(x, y) &= e^y \cos x \\
  f(x, y) &= (x^2 + y^2)((x - 1)^2 - (y - 1)^2)(- (x - 2)^2 - (y - 2)^2)
\end{align*}
\]

34) Use the method of Lagrange multipliers to find the maximum of \( f(x, y) \) on the constraint \( g(x, y) = 0 \):

\[
\begin{align*}
  f(x, y) &= xy, \quad g(x, y) = x^2 + 2y^2 - 6 \\
  f(x, y) &= y^2 - x^2, \quad g(x, y) = x^2 + 4y^2 - 4 \\
  f(x, y, z) &= x^4 + y^4 + z^4, \quad g(x, y, z) = x^2 + y^2 + z^2 - 1
\end{align*}
\]

35) Find the points on the surface \( S = \{z^2 - x^2 - y^2 = 0\} \) that are closest to the point \((4, 2, 0)\).

36) Find the points on the surface \( S = \{x^2 + 4y^2 - z^2 - 4 = 0\} \) where the tangent plane is parallel to the plane \( 2x + y + z - 5 = 0 \).